**CORRECTION KEY Math 536.05**

| Part A | |
|---|---|
| **Questions 1 to 10** | ***4 marks or 0 marks*** |

| | | | |
|---|---|---|---|
| **1** | C | **6** | D |
| **2** | A | **7** | C |
| **3** | B | **8** | D |
| **4** | C | **9** | B |
| **5** | A | **10** | B |

| Part B | |
|---|---|
| **Questions 11 to 15** | ***4 marks each*** |

**11** The coordinates of the foci are **(1, -1)** and **(1, -7)**. *4, 2 or 0 marks* /4
*Allot 2 marks for each ordered pair.*

**12** $f^{-1}(x) = \dfrac{8 + 2x}{x - 1}$ or $f^{-1}(x) = \dfrac{10}{x - 1} + 2$ *4 marks or 0 marks* /4

**13** The equations of the asymptotes are $\boldsymbol{x = -4}$ and $\boldsymbol{y = 1}$. *4, 2 or 0 marks* /4
*Allot 2 marks for each equation.*

**14** Example of an appropriate solution /4

$$2\cos^2\theta - 3\sin\theta = 3$$
$$2\left(1 - \sin^2\theta\right) - 3\sin\theta = 3 \qquad \text{1 } \textit{mark}$$
$$2 - 2\sin^2\theta - 3\sin\theta = 3$$
$$2\sin^2\theta + 3\sin\theta + 1 = 0$$
$$\left(2\sin\theta + 1\right)\left(\sin\theta + 1\right) = 0$$
$$\sin\theta = \frac{-1}{2} \quad \text{or} \quad \sin\theta = -1 \qquad \text{1 } \textit{mark}$$

For $\sin\theta = \frac{-1}{2}$ For $\sin\theta = -1$

Reference angles: $\theta = \frac{\pi}{6}$ $\theta = \frac{3\pi}{2}$ *1 mark*

The sine function is negative in Quadrants III and IV.

Answer: In radian measure, the exact answers are $\theta \in \left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$ *1 mark*

Note: *Correct answers in degree measure, or rounded numerical answers receive a maximum of 2 marks.*

**15** Example of an appropriate solution /4

$$\begin{aligned}
\vec{u}\left(2\vec{u} \bullet 3\vec{v}\right) &= \vec{u}\left(2(-1, 1) \bullet 3(1, 2)\right) \\
&= \vec{u}\left((-2, 2) \bullet (3, 6)\right) && \text{1 } \textit{mark} \\
&= \vec{u}\left((-2 \bullet 3) + (2 \bullet 6)\right) \\
&= \vec{u}\left((-6) + (12)\right) \\
&= \vec{u}(6) && \text{1 } \textit{mark} \\
&= 6\vec{u} \\
&= 6(-1, 1) \\
&= (-6, 6) && \text{2 } \textit{marks}
\end{aligned}$$

Answer: The components of $\vec{u}\left(2\vec{u} \bullet 3\vec{v}\right)$ are **(-6, 6)**.

**Part C**

**Questions 16 to 25** *4 marks each*

**No marks are to be given if work is not shown. Examples of correct solutions are given. However, other acceptable solutions are possible.**

**16** Example of an appropriate solution /4

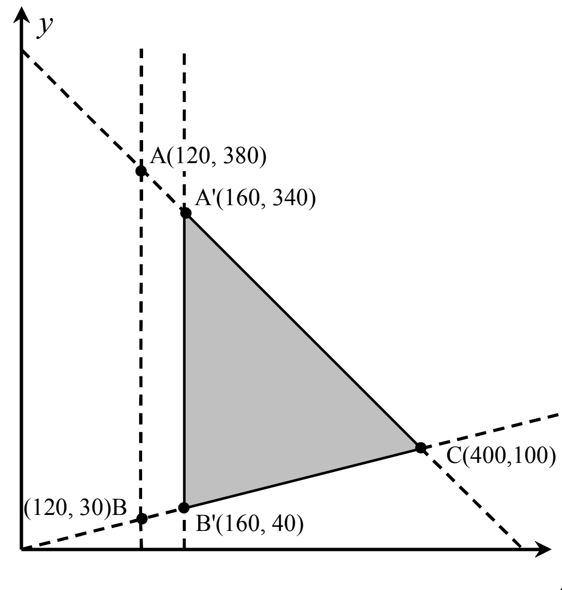
$x$: number of almond chocolate bars
$y$: number of caramel chocolate bars

Constraints

$$x + y \leq 500$$
$$x \leq 4y$$
$$x \geq 120$$

Profit

$$P = 0.8x + y$$




| Point | Profit |
|---|---|
| A(120, 380) | \$476 |
| B(120, 30) | \$126 |
| C(400, 100) | \$420 |

| Point | Profit |
|---|---|
| A'(160, 340) | \$468 |
| B'(160, 40) | \$168 |
| C'(400, 100) | \$420 |

Difference in profit

$476 – $468 = $8

Answer: The difference in the maximum profit is $**8**.

**Note:** Students who use an appropriate method in order to determine the constraints, graph the polygon and find the original corner points have shown they have a partial understanding of the problem.

**17** Example of an appropriate solution /4

Greatest integer function

$$x = 0 \Rightarrow \quad y = 32.5[0.05(0) + 3] + 52.5$$
$$= 32.5[3] + 52.5$$
$$= 150 \text{ cm}$$

Step length $= \dfrac{1}{0.05} = 20 \Rightarrow$ Last open point is (20, 150)

Rational function

$$y = \frac{a}{x + 10} - 10$$
$$150 = \frac{a}{20 + 10} - 10$$
$$160 = \frac{a}{30}$$
$$a = 4800$$

Equation

$$y = \frac{4800}{x + 10} - 10$$

$$x = 180 \Rightarrow \quad y = \frac{4800}{180 + 10} - 10$$
$$\approx 15.26 \text{ cm}$$

Answer: To the nearest tenth of a centimetre, the distance is **15.3** cm.

**Note:** Students who use an appropriate method in order to correctly determine the point (20, 150) have shown they have a partial understanding of the problem.

**18** Example of an appropriate solution /4

Find the equation of the absolute value function

Vertex (1, 10) point (0, 2)

$$y = a|x - 1| + 10$$
$$2 = a|-1| + 10$$
$$-8 = a$$
$$y = -8|x - 1| + 10$$

Find the $x$ value when $y = 1$

$$y = -8|x - 1| + 10$$
$$1 = -8|x - 1| + 10$$
$$-9 = -8|x - 1|$$
$$\frac{9}{8} = |x - 1|$$

$$\therefore \quad x - 1 = \frac{-9}{8} \quad \text{or} \quad x - 1 = \frac{9}{8}$$
$$x = \frac{-1}{8} \qquad\qquad x = \frac{17}{8}$$
$$x = \frac{17}{8} \quad \text{or} \quad 2.125$$

Find the equation of the square root function

Starting point (2.125, 1) point (3.125, 3)

$$y = a\sqrt{x - 2.125} + 1$$
$$3 = a\sqrt{3.125 - 2.125} + 1$$
$$2 = a\sqrt{1}$$
$$2 = a$$

So $y = 2\sqrt{x - 2.125} + 1$

Find the time when the height is 5 m

$$y = 2\sqrt{x - 2.125} + 1$$
$$5 = 2\sqrt{x - 2.125} + 1$$
$$4 = 2\sqrt{x - 2.125}$$
$$2 = \sqrt{x - 2.125}$$
$$4 = x - 2.125$$
$$6.125 = x$$

Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.

**Note:** Students who use an appropriate method in order to determine the starting point of the square root function have shown they have a partial understanding of the problem.

Do not penalize students who rounded their final answer.

**19** Example of an appropriate solution /4

Let $t$: number of days
$f(t)$: amount of the compound remaining (g)

$$f(t) = 150c^t$$
$$123 = 150c^{10}$$
$$0.82 = c^{10}$$
$$c = \sqrt[10]{0.82}$$
$$c \approx 0.98$$

Time for 75 g to remain:

$$f(t) = 150(0.98)^t$$
$$75 = 150(0.98)^t$$
$$0.5 = (0.98)^t$$
$$t = \log_{0.98}(0.5)$$
$$t = \frac{\log(0.5)}{\log(0.98)}$$
$$t \approx 34.3$$

Answer: To the nearest day, half of the compound will remain after **34** days.

**Note:** Students who do not round $\sqrt[10]{0.82}$ will obtain a rounded answer of 35 days. Accept answer of 34 or 35 days if appropriate work is shown.

Students who use an appropriate method in order to correctly determine the value $c \approx 0.98$ have shown they have a partial understanding of the problem.

Students who use the half-life formula $N = N_o(0.5)^{\frac{t}{H}}$ where H is the half-life, and who obtain $\frac{10}{H} = \log_{0.5}(0.82))$ have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

**20** Example of an appropriate solution /4

$$a + c = 167$$
$$2c = 132$$

$\therefore$ $c = 66$ and $a = 101$

$$b^2 = a^2 - c^2$$
$$b = \sqrt{101^2 - 66^2}$$
$$b \approx 76.45$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{10\ 201} + \frac{y^2}{5845} = 1$$
$$\frac{66^2}{10\ 201} + \frac{y^2}{5845} = 1$$
$$y \approx 57.87$$

Answer: To the nearest length, the distance from Mars to the Sun is **57.9** AU.

**Note:** Students who use an appropriate method in order to determine parameters $a$, $b$ and $c$ have shown they have a partial understanding of the problem.

Do not penalize students who did not round their answers or rounded incorrectly.

**21** Example of an appropriate solution /4

**Circle**
Centre: (13, 10) radius: 4 cm

End points of the diameter
(9, 10) and (17, 10)

**Hyperbola**
The vertices are (9, 10) and (17, 10) and therefore $a = 4$.

Half of the total length is 7 cm. So the foci are (6, 10) and (20, 10) and therefore $c = 7$.

Equation of hyperbola

$$c^2 = a^2 + b^2$$
$$7^2 = 4^2 + b^2$$
$$33 = b^2$$

$$\therefore \quad \frac{(x-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$

To find the height let $x = 6$ and find the $y$ coordinate

$$\frac{(6-13)^2}{16} - \frac{(y-10)^2}{33} = 1$$
$$33(-7)^2 - 16(y-10)^2 = 16(33)$$
$$1617 - 528 = 16(y-10)^2$$
$$1089 = 16(y-10)^2$$
$$68.0625 = (y-10)^2$$
$$\pm 8.25 = (y-10)$$

So $y = 10 + 8.25 = 18.25$ and $y = 10 - 8.25 = 1.75$

Answer: The height of the frame is **16.5** cm.

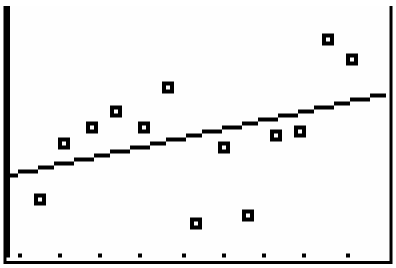
**Note:** Students who use an appropriate method in order to determine a correct equation of the hyperbola have shown they have a partial understanding of the problem.

**22** Example of an appropriate solution /4

Let $x$: year
$y$: number of homicides

Using a graphing calculator

Calculator method



LinReg
$y = ax + b$
$a = 5.631868132$
$b = -10527.04396$
$r^2 = 0.1566628593$
$r = 0.3958065933$

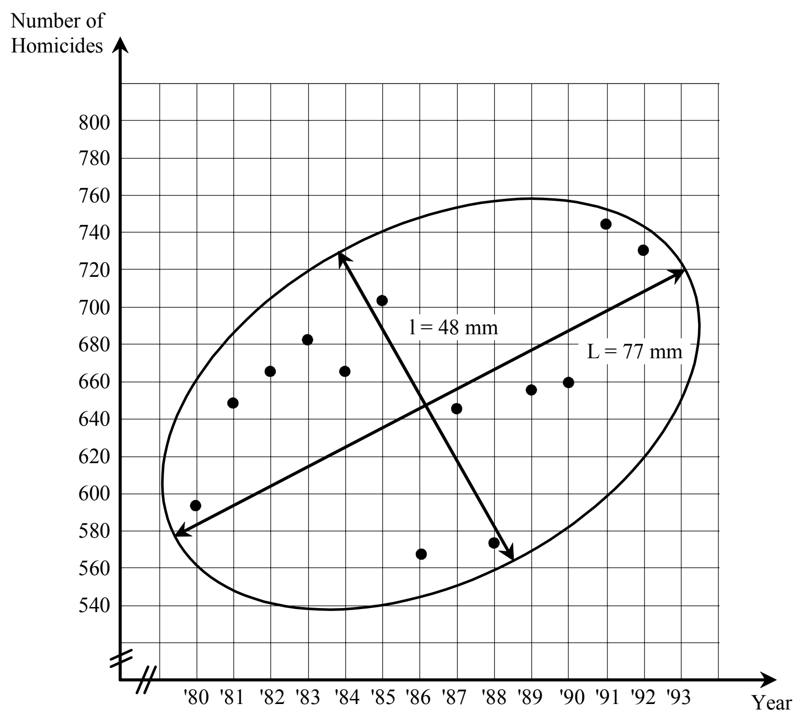
$y_1(2002)$
747.9560403

**Note:** Students may obtain 624.0549451 as the value of parameter $b$ if they use the number of years elapsed since 1980 as the independent variable.

Without a graphing calculator

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{732 - 592}{1992 - 1980}$$
$$= 11.7$$
$$y = 11.7x + b$$
$$732 = 11.7(1992) + b$$
$$b = -22\ 574$$
$$y = 11.7x - 22\ 574$$
$$y = 11.7(2002) - 22\ 574$$
$$y = 849$$
$$r = \left(1 - \frac{48}{77}\right)$$
$$\approx 0.38$$

(Student values will vary)




Therefore, it would be expected that approximately 748 (849 or equivalent using a graph) homicides would occur in 2002.

Answer: Explanation: 582 homicides are not consistent with the model, which would suggest significantly more homicides than actually occurred. Since the correlation coefficient is low (0.40), the model itself is not a reliable predictor.

**Note:** Students who use an appropriate method in order to determine a correct equation for the line of regression have shown they have a partial understanding of the problem.

**23** Example of an appropriate solution /4

| | | |
|---|---|---|
| $\angle ABC = 90°$ | An angle inscribed in a semi-circle is a right angle | |
| Length of $\overline{AC}$ | $5(m\ \overline{AC}) = 13^2$<br>$5(m\ \overline{AC}) = 169$<br>$m\ \overline{AC} = 33.8$ cm | proportional mean |
| Length of $\overline{ED}$ | $m\ \overline{ED} + 14 + 5 = 33.8$<br>$m\ \overline{ED} = 14.8$ cm | length of the diameter |
| Length of $\overline{EB}$ | $m\ \overline{EB}^2 = 12^2 + 14.8^2$<br>$m\ \overline{EB} = 19.05$ cm | |
| Length of $\overline{EF}$ | $m\ \overline{EF}(19.05) = 14(33.8 - 14)$<br>$m\ \overline{EF}(19.05) = 14(19.8)$<br>$m\ \overline{EF} = 14.55$ (14.548 if you use 19.053 608 58) | constant product theorem |

Answer: To the nearest tenth, the measure of $\overline{EF}$ is **14.6** cm.

**Note:** Students who use an appropriate method in order to determine the length of the diameter have shown they have a partial understanding of the problem.

Do not penalize students who did not round their answers or rounded incorrectly.

**24** Example of an appropriate solution /4

$$\left(m\ \overline{AB}\right)^2 = \left(m\ \overline{AF}\right) \times \left(m\ \overline{AE}\right)$$
$$16^2 = 8\ m \times \left(m\ \overline{AE}\right)$$
$$m\ \overline{AE} = 32\ cm$$

Segment $\overline{DE}$ is a tangent and segment $\overline{OE}$ is a radius, so m $\angle$ DEA = 90°.

In $\Delta$ AED

$$\left(m\ \overline{AD}\right)^2 = \left(m\ \overline{AE}\right)^2 + \left(m\ \overline{DE}\right)^2$$
$$\left(m\ \overline{AD}\right)^2 = 32^2 + 24^2$$
$$\left(m\ \overline{AD}\right)^2 = 1600$$
$$m\ \overline{AD} = 40\ cm$$

$$\left(m\ \overline{AD}\right)\left(m\ \overline{EC}\right) = \left(m\ \overline{DE}\right)\left(m\ \overline{AE}\right)$$
$$40\left(m\ \overline{EC}\right) = (24)(32)$$
$$m\ \overline{EC} = \frac{768}{40}$$
$$m\ \overline{EC} = 19.2\ cm$$

Answer: The measure of $\overline{EC}$ is **19.2** cm.

**Note:** Students who use an appropriate method in order to correctly determine $m\ \overline{AE}$ have shown they have a partial understanding of the problem.

**25** Example of an appropriate solution /4

The minimum height of the pendulum is 150 cm.

The maximum height of the pendulum is 150 + (31 – 31 sin 45°) = 159.08 cm.

$$y = a \cos b(x - h) + k$$

$$a = \frac{159.08 - 150}{2}$$
$$= 4.54$$

$$b = \frac{2\pi}{0.875}$$
$$= \frac{16\pi}{7}$$

$$k = \frac{159.08 + 150}{2}$$
$$= 154.54$$

$h = 0$ if $a$ is negative

$$f(t) = -4.54 \cos\left(\frac{16\pi t}{7}\right) + 154.54$$

1 hour = 3600 seconds
$f(3600) = 156$ cm

Answer: To the nearest cm, the pendulum will be **156** cm above the ground after 1 hour.

**Note:** Students who use an appropriate method in order to determine parameters $a$, $b$ and $k$ have shown they have a partial understanding of the problem.

Students' answers may vary considerably, depending on the rounding they applied in the course of