3. GUIDELINES

- Hand out the Question Booklets and Answer Booklets.
- Read the instructions that appear in the Question Booklet aloud.

4. CORRECTION KEY

Section A

Questions 1 to 10 *4 marks or 0 marks* /40

1	A	6	D
2	C	7	A
3	B	8	D
4	B	9	B
5	C	10	C

Section B

11 $x \in\ ]-\infty, -6] \cup [4, +\infty[$ *4 marks or 0 marks* /4
Accept other forms of the answer.

12 The point of intersection is (3, 5). *Give 2 marks for the abscissa and 2 marks for the ordinate.* /4

13 There will be 13 570 insects after **24** weeks. *4 marks or 0 marks* /4

14 Vincent's test mark is 70%. *4 marks or 0 marks* /4

15 *Give 1 mark per correct answer.* /4

1. C
2. B
3. A
4. D

Section C

16 Example of an appropriate solution /4

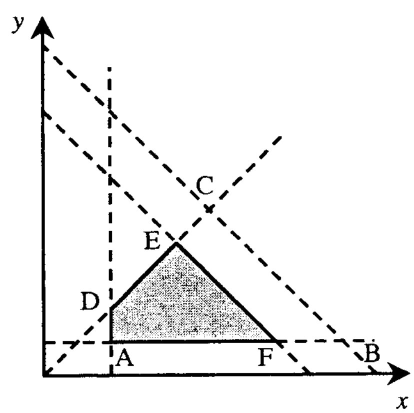
Calculation of revenue before the new constraint is considered

Vertices	$R(x, y) = 1.00x + 1.50y$
A(100, 50)	100 + 75 = \$175
B(450, 50)	450 + 75 = \$525
C(250, 250)	250 + 375 = \$625
D(100, 100)	100 + 150 = \$250

Maximum Revenue ⇒ \$625 250 cases of oranges and 250 cases of grapefruit

Calculation of revenue after consideration of the constraint $x + y \leq 400$

Vertices	$R(x, y) = 1.00x + 1.50y$
A(100, 50)	\$175
D(100, 100)	\$250
E(200, 200)	\$500
F(350, 50)	\$425



Maximum Revenue ⇒ 500 \$ 200 cases of oranges 200 cases of grapefruit

Decrease of revenue because of the flood

625 – 500 = 125

Answer: The decrease in revenue caused by the flood is **\$125**.

17 Example of an appropriate solution /4

Let x: time measured in weeks
$f(x)$: percentage of popular vote

Coordinates of points
P(0, 28)
S(10, 43)

Find the equation
$$f(x) = a|x - h| + k$$
$$f(x) = a|x - 10| + 43 \qquad \text{domain } f = [0, 26]$$

Substitute P(0, 28) into the equation
$$28 = a|0 - 10| + 43$$
$$-15 = a \bullet 10$$
$$-1.5 = a$$

$$f(x) = -1.5|x - 10| + 43$$

Solve the equation
$$f(x) = 25$$
$$-1.5|x - 10| + 43 = 25$$
$$-1.5|x - 10| = -18$$
$$|x - 10| = 12$$

$|x - 10| = 12$
$x - 10 = 12$ or $-x + 10 = 12$
$x = 22$ $\quad -x = 2$
$x = -2$ before the poll

Answer: Party "A" has one quarter of the popular vote at **22** weeks.

18 /4

Example of an appropriate solution

Let x: number of copies sold
y: profit per copy sold

Find the rule of correspondence

$$\begin{aligned} y &= \frac{14x - 5x - 20\,000}{x} \\ &= \frac{9x - 20\,000}{x} \end{aligned}$$

Solve for a profit of \$4 per copy

$$\begin{aligned} 4 &= \frac{9x - 20\,000}{x} \\ 4x &= 9x - 20\,000 \\ 20\,000 &= 5x \\ 4000 &= x \end{aligned}$$

Answer: The publisher must sell **4000** copies in order to make a profit of \$4 per book.

19 /4

Example of an appropriate solution

Equation of the square root function

Vertex (0, 2,4)
Point (16, 0)

$$\begin{aligned} f(x) &= A\sqrt{b(x-h)} + k \qquad\qquad b = 1 \\ 0 &= A\sqrt{1(16-0)} + 2.4 \\ 0 &= a\sqrt{16} + 2.4 \\ 0 &= 4a + 2.4 \\ -2.4 &= 4a \\ -0.6 &= A \end{aligned}$$

So, $f(x) = -0.6\sqrt{x} + 2.4$

Coordinates of the point where she stops $(10, h)$

$$\begin{aligned} f(x) &= -0.6\sqrt{x} + 2.4 \\ h &= -0.6\sqrt{10} + 2.4 \\ h &\approx 0.5 \end{aligned}$$

Answer: Carol stops at a height of **0.5** m.

20 Example of an appropriate solution /4

Let t: time, in minutes, which has past since 21:00
$A(t)$: altitude of the airplane after t minutes

The rule of correspondence

$$A(t) = a \times c^t$$

$$\begin{aligned} 10\,000 &= a \times c^0 \\ 10\,000 &= A \\ A(t) &= 10\,000c^t \\ 5222 &= 10\,000c^4 \\ 0.5222 &= c^4 \\ (0.5222)^{\frac{1}{4}} &= c \\ 0.85 &\approx c \end{aligned}$$

$P_1(0, 10\,000)$
$P_2(4, 5222)$

$$A(t) = 10\,000(0.85)^t$$

t when $A(t) = 280$

$$\begin{aligned} 280 &= 10\,000(0.85)^t \\ 0.028 &= 0.85^t \\ t &= \log_{0.85} 0.028 \\ t &= \frac{\log 0.028}{\log 0.85} \\ t &\approx 22 \end{aligned}$$

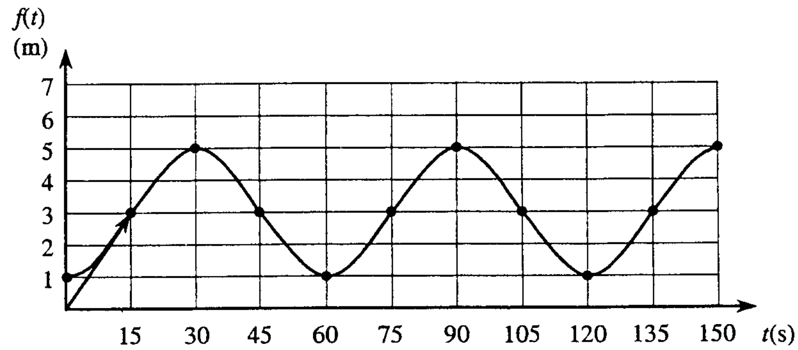
Answer: The airplane will be at an altitude of 280 m at **21:22**.

21 Example of an appropriate solution /4

Let t: time, in seconds, that has past since 13:00
$f(t)$: height of the jet, in metres

The rule of correspondence
$f(t) = a \sin b(t - h) + k$

$$a = \frac{5-1}{2} = 2$$
$$p = \frac{2\pi}{|b|}$$
$$60 = \frac{2\pi}{|b|}$$
$$|b| = \frac{2\pi}{60}$$
$$|b| = \frac{\pi}{30}$$



$$f(x) = 2 \sin \frac{\pi}{30}(t - h + k)$$

Translation $(h, k) = (15, 3)$.
$$f(t) = 2 \sin \frac{\pi}{30}(t - 15) + 3$$

Height at 13 h 12 min 40 s
Since the function has a period of one minute, the jet will be at the same height in 40 seconds.

$$f(40) = 2 \sin \frac{\pi}{30}(40 - 15) + 3$$
$$\approx 3.09$$

Answer: At 13 hours 12 minutes and 40 seconds, the water jet will be at a height of **4** m.

22 Example of an appropriate solution /4

$$\tan x + \frac{\cos x}{1 + \sin x} = \sec x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \sec x \quad ①$$

$$\frac{\sin x(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)} = \sec x$$

$$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} = \sec x \quad ②$$

$$\frac{\sin x + 1}{\cos x(1 + \sin x)} = \sec x \quad ③$$

$$\frac{1}{\cos x} = \sec x \quad ④$$

$$\sec x = \sec x$$

① *1 mark*
② *1 mark*
③ *1 mark*
④ *1 mark*

23 Example of an appropriate solution /4

Equation of the hyperbola

Form $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

Distance between the vertices

$20 - -4 = 24$

Since the center of the hyperbola is C(8, 0), $h = 8$ and $k = 0$

Value of a

$$a = \frac{24}{2} = 12$$

Value of b

The equation of an asymptote is $y = 2(x - 8)$

$$\frac{b}{a} = 2$$

$$\frac{b}{12} = 2$$

$$b = 24$$

$$\therefore \frac{(x-8)^2}{144} - \frac{y^2}{576} = 1$$

Coordinates of points P and S

Abscissa = radius of the circle = $\sqrt{900} = 30$

Ordinates

$$\frac{(30-8)^2}{144} - \frac{y^2}{576} = 1$$

$$\frac{22^2}{144} - \frac{y^2}{576} = 1$$

$$\frac{484}{144} - 1 = \frac{y^2}{576}$$

$$y^2 = 576 \times \frac{340}{144}$$

$$y \approx \pm 36.9$$

P(30, 37)
S(30, -37)

Distance between the traffic signs

$\text{m}\,\overline{PS} = 37 - -37$

$= 74$

Answer: The distance between the two signs is **74** m.

24 Example of an appropriate method /4

Equation of the tangent line

$$4x + 3y - 43 = 0 \quad \text{or} \quad y = \frac{-4}{3}x + \frac{43}{3}$$

Equation of the perpendicular line passing through C(3, 2)

$$y = \frac{3}{4}x - \frac{1}{4}$$

Intersection of the two lines

$$\frac{-4}{3}x + \frac{43}{3} = \frac{3}{4}x - \frac{1}{4}$$

$$\frac{25}{12}x = \frac{175}{12}$$

$$x = 7$$

If $x = 7$, $y = \frac{3}{4}(7) - \frac{1}{4} = 5$

The coordinate at the point of intersection is P(7, 5).

Measure of the radius of the circle

$$d(C, P) = \sqrt{(7-3)^2 + (5-2)^2} = \sqrt{25} = 5$$

Equation of the circle $(x - 3)^2 + (y - 2)^2 = 25$

Answer The equation of the circle is $\mathbf{(x - 3)^2 + (y - 2)^2 = 25}$.

The general form of the equation, $x^2 + y^2 - 6x - 4y - 12 = 0$, should also be accepted.

25 Example of an appropriate solution /4

Calculate the length of segment $\overline{AB}$, given triangle ABC is isosceles and therefore $\overline{AM} = 12$:

$$\begin{aligned}\overline{AB} &= \sqrt{\left(\left(\overline{AM}\right)^2 + \left(\overline{BM}\right)^2\right)} \\ &= \sqrt{12^2 + 9^2} \\ &= \sqrt{225} \\ &= 15 \text{ dm}\end{aligned}$$

Calculate the length of segment $\overline{PM}$

$$\overline{PM} \cdot \overline{AB} = \overline{AM} \cdot \overline{BM}$$

(In a right triangle, the length of the hypotenuse multiplied by the length of the altitude to the hypotenuse is equal to the product of the lengths of the sides of the right angle.)

$$\begin{aligned}\overline{PM} \bullet 15 &= 12 \bullet 9 \\ \overline{PM} &= 7.2 \text{ dm}\end{aligned}$$

Answer: Segments $\overline{PM}$ and $\overline{QM}$ each measure 7.2 dm.