Math 536: June…Answers & Solutions

CORRECTION KEY

Part A	
Questions 1 to 8	***4 marks or 0 marks***

1	B	**5**	B
2	C	**6**	C
3	A	**7**	D
4	C	**8**	A

Part B	
Questions 9 to 15	***4 marks each***

9 The range of $f^{-1}(x)$ is $-\infty, -5]$ *4 or 0 marks* /4

10 The domain of $f(x)$ is $]-\infty, 3[\cup]3, +\infty[$. *4, 2 or 0 marks* /4
The range of $f(x)$ is $]-\infty, -1.5[\cup]-1.5, +\infty[$.
Assign 2 marks for each answer.

11 The solution set is $x = 4$ *4, 2 or 0 marks* /4
Assign 2 marks if the student included the extraneous root, -2.5.

12 The measure of ∠ DCE is **35°**. *4, 3 or 0 marks* /4

Assign 3 marks if the student answered 70°, having forgotten to divide by 2.

13 To the nearest unit, the magnitude is **8** units. *4, 2 or 0 marks* /4

To the nearest degree, the direction is **W76°N** or equivalent.

Assign 2 marks for each answer.

14 The exact values of x are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$. *4, 2 or 0 marks* /4

15 Example of an appropriate solution /4

$$\frac{\sec\theta}{\tan\theta + \cot\theta} \equiv \sin\theta$$

$$\frac{\sec\theta}{\tan\theta + \cot\theta}$$

$$\frac{\sec\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$ 1 mark

$$\frac{\sec\theta}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}$$ 1 mark

$$\frac{\sec\theta}{\frac{1}{\cos\theta\sin\theta}}$$ 1 mark

$$\frac{1}{\cos\theta} \times \cos\theta\sin\theta$$

$$\sin\theta$$ 1 mark

Part C

Questions 16 to 25 ***4 marks each***

No marks are to be given if work is not shown. Examples of correct solutions are given. However, other acceptable solutions are possible.

16 Example of an appropriate solution /4

x: number of hours at first job per month
y: number of hours at second job per month

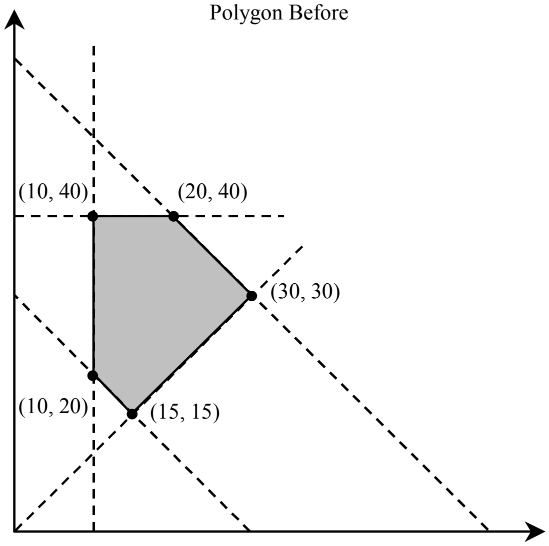
Constraints before

$x \geq 10$
$y \leq 40$
$y \geq 0$
$x + y \geq 30$
$x + y \leq 60$
$y \geq x$

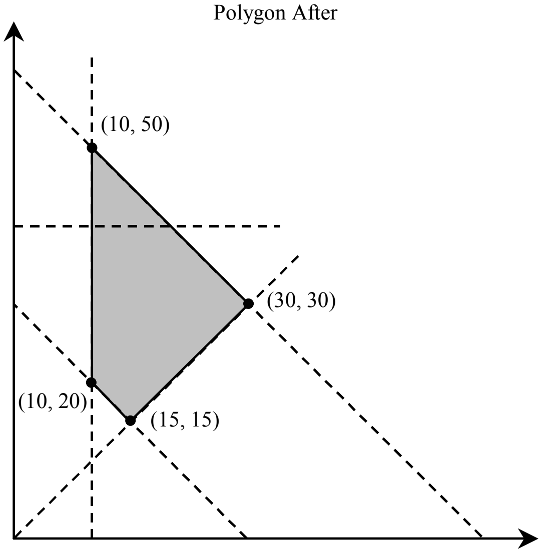
Constraints after

$x \geq 10$
$y \geq 0$
$x + y \geq 30$
$x + y \leq 60$
$y \geq x$

Polygon Before



Polygon After



Maximum Before

Vertices	S = 6.3x + 8y (\$)
A(10, 40)	383
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
E(20, 40)	446

Maximum After

Vertices	S = 6.3x + 8y (\$)
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
F(10, 50)	463

Difference in maximum salary
\$463 – \$446 = \$17

Answer: Murray's maximum possible salary increased by \$17

Note: Students who use an appropriate method in order to determine **the constraints, graph the original polygon, and find its vertices** have shown they have a partial understanding of the problem.

17 Example of an appropriate solution /4

Vertex is (80, 140) and the initial value is (0, 40).

Rate of change of the left arm

$$\frac{140 - 40}{80 - 0} = \frac{100}{80} = \frac{5}{4}$$

The rate of the left arm is $\frac{5}{4}$, therefore the rate of the right arm is $-\frac{5}{4}$.

Hence the rule

$$H(t) = -\frac{5}{4}\left|t - 80\right| + 140$$

Substitute $H(t) = 60$

$$60 = -\frac{5}{4}\left|t - 80\right| + 140$$
$$-80 = -\frac{5}{4}\left|t - 80\right|$$
$$64 = \left|t - 80\right|$$
$$\left|t - 80\right| = 64$$

For $t - 80 \geq 0, t \geq 80$

$$\left|t - 80\right| = 64$$
$$t - 80 = 64$$
$$t = 144$$

For $t - 80 \leq 0, t \leq 80$

$$\left|t - 80\right| = 64$$
$$t - 80 = -64$$
$$t = 16$$

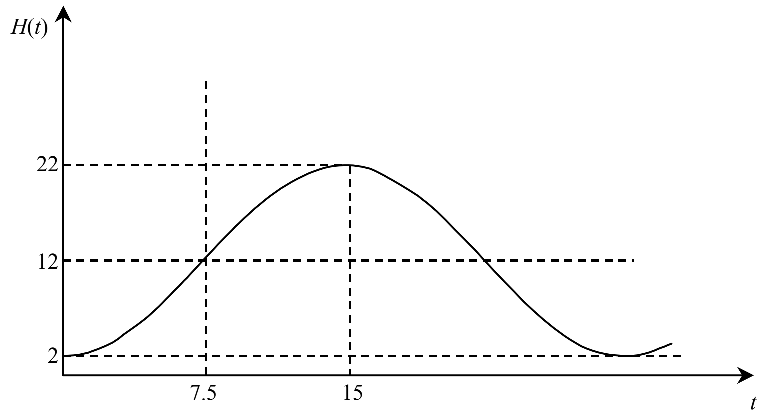
Answer: The helicopter was at an altitude of 60 m after **16** seconds as it ascended and **144** seconds as it descended.

Note: Students who have found **the rule of the absolute value** have shown they have a partial understanding of the problem.

Once students have found the rule, they can use a graphing calculator to determine the intersection between the absolute value and the line $y = 60$. Appropriate explanation must be provided.

18 Example of an appropriate solution /4

Rule of the function



t: time in seconds
$H(t)$: height in metres

$$H(t) = a \sin b(x - h) + k$$

$$|a| = \frac{22 - 2}{2} = 10$$

$$H(t) = 10 \sin b(x - 7.5) + 12$$

$$\frac{1}{2}(\text{period}) = 15, \ \text{P} = 30 \text{ seconds}$$

$$30 = \frac{2\pi}{|b|}$$

$$|b| = \frac{2\pi}{30} = \frac{\pi}{15}$$

$$H(t) = 10 \sin \frac{\pi}{15}(x - 7.5) + 12$$

Height of Tom's seat after 20 seconds

$$H(20) = 10 \sin \frac{\pi}{15}(20 - 7.5) + 12$$
$$= 17 \text{ metres}$$

Answer: Tom's seat is **17** m above the ground 20 seconds after the Ferris wheel begins to turn.

Note: $H(t) = \text{-}10 \cos \frac{\pi}{15}x + 12$ and $H(t) = \text{-}10 \sin \frac{\pi}{15}(x - 22.5) + 12$ are examples of other acceptable rules.

Note: Students who use an appropriate method in order to determine **parameters *a* and *b*** have shown they have a partial understanding of the problem.

19 Example of an appropriate solution /4

$2c = 100 \qquad c = 50$
$2a = 60 \qquad a = 30$

$$\therefore \; b^2 = c^2 - a^2$$
$$b = \sqrt{2500 - 900}$$
$$b = 40$$

Equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\therefore \; \frac{x^2}{900} - \frac{y^2}{1600} = 1$$

Sub in $x = 50$

$$\frac{50^2}{900} - \frac{y^2}{1600} = 1$$
$$y^2 = 1600\left(\frac{1600}{900}\right)$$
$$y \approx \pm 53.3$$

Height

2×53.3 cm ≈ 106.6 cm

Answer: Height PQ of the leg of the table saw is approximately **106.6** cm.

Note: Students who have determined **the equation of the hyperbola** have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

20 Example of an appropriate solution /4

$$m\,\overline{AD} \times m\,\overline{DB} = m\,\overline{CD} \times m\,\overline{DE}$$
$$8 \text{ cm} \times 10 \text{ cm} = m\,\overline{CD} \times 5 \text{ cm}$$
$$m\,\overline{CD} = \frac{8 \text{ cm} \times 10 \text{ cm}}{5 \text{ cm}}$$
$$m\,\overline{CD} = 16 \text{ cm}$$

$$m\,\overline{PC} \times m\,\overline{PE} = \left(m\,\overline{PT}\right)^2$$
$$4 \text{ cm} \times 25 \text{ cm} = \left(m\,\overline{PT}\right)^2$$
$$\left(m\,\overline{PT}\right)^2 = 100 \text{ cm}^2$$
$$m\,\overline{PT} = 10 \text{ cm}$$

Segment PT is a tangent and segment OT is a radius. $\therefore$ m $\angle$ PTO = 90°

In Δ PTO

$$\left(m\,\overline{OT}\right)^2 = \left(m\,\overline{OP}\right)^2 - \left(m\,\overline{PT}\right)^2$$
$$\left(m\,\overline{OT}\right)^2 = (17 \text{ cm})^2 - (10 \text{ cm})^2$$
$$m\,\overline{OT} = \sqrt{289 \text{ cm}^2 - 100 \text{ cm}^2}$$
$$m\,\overline{OT} \approx 13.74$$

Answer: To the nearest tenth, the measure of the radius is **13.7** cm.

Note: Students who use an appropriate method in order to determine $m\,\overline{CD}$ have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

21 Examples of appropriate solutions /4

Example 1

Find m $\overline{AB}$ by using right triangle/mean proportion relation
Δ ABC is right angled at A

$$\left(m\ \overline{AB}\right)^2 = m\ \overline{BF} \times m\ \overline{BC}$$
$$\left(m\ \overline{AB}\right)^2 = 9(24)$$
$$\left(m\ \overline{AB}\right)^2 = 216$$
$$m\ \overline{AB} = 6\sqrt{6}\ \text{cm}$$

Δ ABF ~ Δ AED and m $\overline{AF}$ = m $\overline{FD}$

$$\therefore \quad \frac{m\ \overline{AF}}{m\ \overline{AD}} = \frac{m\ \overline{AB}}{m\ \overline{AE}} = \frac{1}{2}$$

$$m\ \overline{AE} = 2\ m\ \overline{AB} = 12\sqrt{6}\ \text{cm}$$

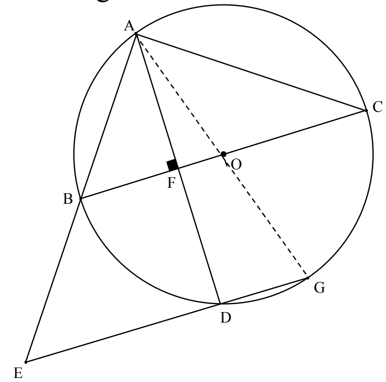
$$\frac{m\ \overline{BF}}{m\ \overline{ED}} = \frac{1}{2}$$
$$m\ \overline{BF} = 9\ \text{cm}$$
$$\therefore \quad m\ \overline{ED} = 18\ \text{cm}$$

$\overline{EA}$ and $\overline{EG}$ are secants.
Let x = m $\overline{DG}$

$$\therefore \quad m\ \overline{EB} \times m\ \overline{EA} = m\ \overline{ED} \times m\ \overline{EG}$$
$$\left(6\sqrt{6}\right)\left(12\sqrt{6}\right) = 18(18 + x)$$
$$432 = 18(18 + x)$$
$$24 = 18 + x$$
$$6 = x$$

Example 2

Join points A and G through O



Since Δ ABF ~ Δ AED

∴ $\overline{BC}$ // $\overline{EG}$ since m ∠ AFB = m ∠ ADE

Since $\overline{BC}$ bisects $\overline{AD}$ and $\overline{BC}$ // $\overline{EG}$,
$\overline{BC}$ also bisects $\overline{AG}$

$$\therefore \quad \frac{m\ \overline{AD}}{m\ \overline{AF}} = \frac{m\ \overline{AG}}{m\ \overline{AO}} = \frac{2}{1}$$

∴ Δ AFO ~ Δ ADG by SAS

$$\therefore \quad \frac{m\ \overline{AD}}{m\ \overline{AF}} = \frac{m\ \overline{DG}}{m\ \overline{FO}} = \frac{2}{1}$$
$$\frac{x}{3\ \text{cm}} = \frac{2}{1}$$
$$x = 6$$

Answer: To the nearest centimetre, the length of segment DG is **6** cm.

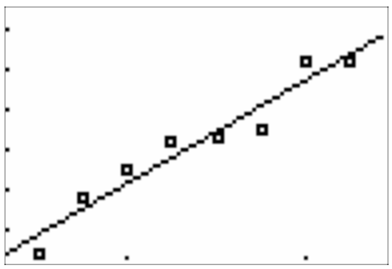
Note: Students who use an appropriate method in order to determine **m $\overline{AE}$ or the m $\overline{ED}$ (example 1)** have shown they have a partial understanding of the problem.

22 Example of an appropriate solution /4

Let x: Force (N)
y: Extension (cm)

Using a graphing calculator

Calculator method



LinReg
$y = ax + b$
$a = 2.572380952$
$b = -.5342857143$
$r^2 = .9369362879$
$r = .9679546931$

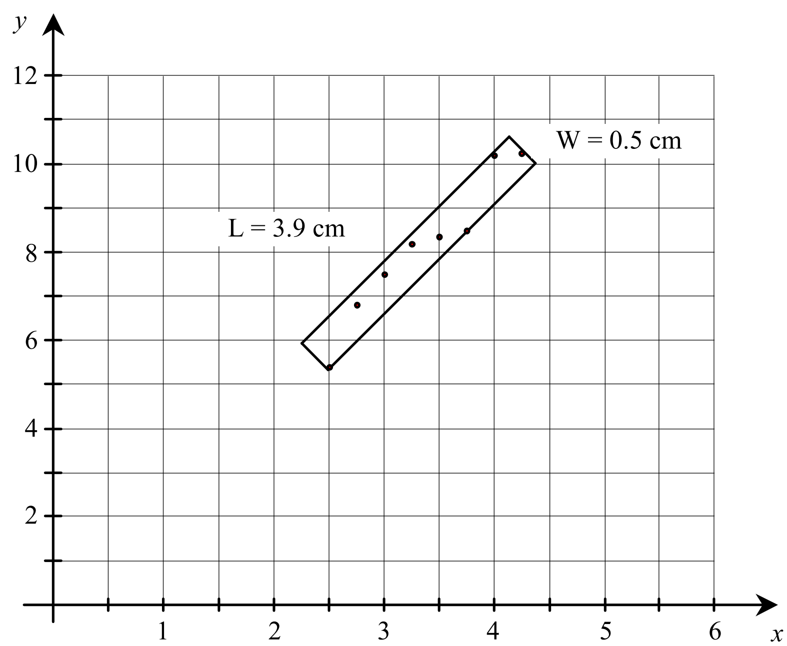
$y_1(6.25)$
15.5430952

Without a graphing calculator

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10.24 - 8.34}{4.25 - 3.5}$$
$$= 2.53$$
$$y = ax + b$$
$$10.24 = 2.53(4.25) + b$$
$$b = -0.5125$$

$$y = 2.53x - 0.5125$$
$$y = 2.53(6.25) - 0.5125$$
$$y = 15.3$$
$$r = \left(1 - \frac{0.5}{3.9}\right)$$
$$\approx 0.87$$

(Student answers may vary.)



According to the data, a force of 6.25 N would seem to be consistent with an extension of 15.54 cm (≈ 15.3 cm using the graph).

Answer: Justification: 15.54 cm is a valid prediction based on the data provided. Since the correlation is very high, the prediction can be made with a high degree of confidence.

Note: Students who have used an appropriate method to determine **the regression equation** have shown they have a partial understanding of the problem.

23 Examples of appropriate solutions /4

Example 1

Let $v(t)$ be the value of Albert's investment t years after 1991

$$v(t) = 4000(\text{base})^t \quad \text{therefore in 1991,}$$
$$4000(c)^8 = 5474.28$$
$$c^8 = 1.368\ 57$$
$$c = \sqrt[8]{1.368\ 57}$$
$$\approx 1.04$$

Time to triple investment

$$4000(1.04)^t = 12\ 000$$
$$(1.04)^t = 3$$
$$t = \frac{\log 3}{\log 1.04}$$
$$\approx 28 \text{ years}$$

Let v_o be the value of Jocelyn's initial investment

$$15\ 000 = v_o\ (1.04)^{28}$$
$$v_o = \left(\frac{15\ 000}{(1.04)^{28}}\right)$$
$$\approx 5002.16$$

Difference between both initial investments

$5002 – $4000 = $1002

Example 2

$$y = ab^x$$

$\frac{y}{a} = b^x$ since the length of time and the rate are both the same.

$\therefore\ \frac{y}{a} = \text{constant}$

$\therefore$ Albert's investment triples in the same length of time that Jocelyn's investment does.

$$\frac{y_1}{a_1} = \frac{y_2}{a_2}$$
$$\frac{12\ 000}{4000} = \frac{15\ 000}{a_2}$$
$$a_2 = \$5000$$

Difference between both initial investments

$5000 – $4000 = $1000

Answer: The difference between Albert's and Jocelyn's initial investments is $**1002**.

Note: Accept answers in the range of $1000 to $1002, as a result of rounding differences.

Students who use an appropriate method in order to correctly determine **the value *c* ≈ 1.04 (example 1)** have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

24 Example of an appropriate solution /4

$$\frac{(x-10)^2}{100} + \frac{(y-8)^2}{64} = 1$$

Centre: (10, 8), $a = 10$ and $b = 8$

$$\therefore \quad c^2 = a^2 - b^2$$
$$c = \sqrt{10^2 - 8^2}$$
$$= 6$$

The coordinates of the foci are: $F_1(10 - 6, 8)$ and $F_2(10 + 6, 8)$

$\therefore$ $F_1(4, 8)$ and $F_2(16, 8)$

Point on square root function, vertex F_2, is equal to center of circle B(12, 11).

Rule of square root function, F_2

$$f(x) = a\sqrt{-(x-h)} + k$$

Vertex: (16, 8)

$$f(x) = a\sqrt{-(x-16)} + 8$$

Substitute in (12, 11) and solve for a

$$11 = a\sqrt{-(12-16)} + 8$$
$$3 = a\sqrt{4}$$
$$a = 1.5$$

$$\therefore \quad f(x) = 1.5\sqrt{-(x-16)} + 8$$

Answer: The rule of the square root function whose vertex is F_2 is $\boldsymbol{f(x) = 1.5\sqrt{-(x-16)} + 8}$.

Note: Students who have determined **F_2 of the ellipse and the center of circle B** have shown they have a partial understanding of the problem.

Deduct one mark if students forgot to put the negative sign in solving for F_2.

25 Example of an appropriate solution /4

Equation of $f(x)$

$$f(x) = a|x - 3| + k$$

Slope of right ray of $f(x)$

$$\frac{-9 - 0}{6 - 4.5} = -6$$

$$f(x) = -6|x - 3| + k$$
$$0 = -6|4.5 - 3| + k \qquad \therefore f(x) = -6|x - 3| + 9$$
$$k = 9$$

Equation of left ray of $f(x)$

$$f(0) = -6|0 - 3| + 9$$
$$= -9$$

$$y = 6x - 9$$

Equation of left ray of $g(x)$

$$g(0) = 4|0 - 3| + 3$$
$$= 15$$

$$y = -4x + 15$$

Coordinates of intersection point

$$6x - 9 = -4x + 15 \qquad y = 6(2.4) - 9$$
$$x = 2.4 \qquad y = 5.4$$

Point A(2.4, 5.4) by symmetry
Point B(3.6, 5.4)

A graphing calculator can also be used to find points A and B. However, students would have to explain the process they had applied.

Area of triangle

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$
$$= \frac{(3.6 - 2.4) \times (9 - 5.4)}{2}$$
$$\text{Area} = \frac{(1.2) \times (3.6)}{2}$$
$$= 2.16$$

Answer: The area of the shaded triangular region is **2.16** units^2.

Note: Do not penalize students who did not round their final answer or rounded incorrectly.

Students who have used an appropriate method to determine **the rule for *f*(*x*)** have shown they have a partial understanding of the problem.