

Mathematics

Comprehensive Exam Number 55

GUIDE

Secondary 5

September, 2005



Youth Sector General Education

1. (GENEI	KAL INFORMATION	
	1.1	Program	Mathematics, Secondary 5 (568-526)
	1.2	Origin	Mathematics and Science & Technology Committee, 2005
			Computerization by Marie-Josée Mickel, BIM, Société GRICS
-	1.3	Time allotted	3 hours
	1.4	Number of questions	25 questions distributed as follows 10 multiple-choice questions 5 short-answer questions 10 developed-response questions
	1.5	Authorized material	 Ruler, compass, set square, protractor Graph paper Scientific calculator with or without a graphic display The calculator must be portable and designed primarily to perform mathematical calculations. Computers and calculators with a QWERTY keyboard, symbol manipulation capabilities or an electronic date book are not permitted. User guides, memory expansion features or any other calculator accessories are not permitted during the examination. Students may not share their calculators with other students. Communication links between calculators are also forbidden during the examination. Memory aid The memory aid is one letter-size sheet of paper (8½ × 11") on which a student will have recorded information of his or her choice. Students are encouraged to work on and revise their memory aid throughout the year. Both sides of the sheet may be used. Any mechanical reproduction of this memory aid is



2. DESCRIPTION OF THE EXAM

The chart below shows the distribution of the items taking into account the relative importance given to the different components of the program.

Themes	Algebra 68%	Geometry 20%	Statistics 12%
Mastery of Concepts 28%	1, 4, 5, 6	9, 10	8
Mastery of Application 40%	2, 3, 7, 12, 13 18, 19	11, 16,	14
Mastery of Problem Solving Techniques 32%	20, 21, 22 23, 24, 25	17	15

Exam Specifications

Note The numbers in the centre of each box represent the question numbers in the examination.

Item Specifications

Question	Item	Objective	S	Т	D
Part A					
1	2076	ALG.01.01	С	Μ	Е
2	2077	ALG.02.04	А	М	Е
3	2078	ALG.02.04	А	Μ	Μ
4	2079	ALG.03.03	С	Μ	E
5	2080	ALG.04.01	С	М	Μ
6	2081	ALG.04.02	С	М	Е
7	2082	ALG.02.03	А	М	D
8	2083	STA.01.06	С	М	Е
9	2084	GEO.01.02	С	М	Μ
10	2085	GEO.01.02	С	М	Е
Part B					
11	2086	GEO.01.02	А	С	Μ
12	2087	ALG.02.04	А	С	Μ
13	2088	ALG.03.05	А	С	Μ
14	2089	STA.01.02	А	С	Μ
15	2090	STA.01.02	Р	С	М
Part C					
16	2091	GEO.01.02	А	E	М
17	2092	GEO.01.02	Р	Е	Μ
18	2093	ALG.01.03	А	Е	Μ
19	2094	ALG.03.03	А	E	Μ
20	2095	ALG.03.04	А	E	Μ
21	2096	ALG.02.05	Р	Е	Μ
22	2097	ALG.02.05	Р	Е	D
23	2098	ALG.04.02	Р	E	Μ
24	2099	ALG.04.03	Р	E	D
25	2100	ALG.04.04	Р	Е	D

Legend

S: Skill	A: Application C: Concept P: Problem solving
T: Type of items	C: Short-constructed answer E: Extended answer (developed response) M: Multiple choice
D: Level of Difficulty	E: Easy M: Medium D: Difficult



INSTRUCTIONS FOR TEACHERS 3.

- Ensure that each student has all the material needed. •
- Hand out the Question Booklets and read the instructions aloud to the students. •
- Collect all booklets at the end of the examination. •

CORRECTION KEY 4.

	Part A		
	Questions 1 to 104 marks or 0 marks		
1	A 6 D		
2	B 7 A		
3	C 8 B		
4	C 9 C		
5	A 10 B		
	Part B		
	Questions 11 to 154 marks each		
11	To the nearest m^2 , the garden area covered in grass is 85 . Do not penalize students who have not rounded or rounded incorrectly.	4 marks or 0 marks	/4
12	The <i>x</i> -intercepts are 2.5 and 7.5 .	4, 2 or 0 marks	/4
13	The coordinates of point P are (0.81, -0.59).	4, 2 or 0 marks	/4
14	 a) Kayla's class mean is 70. b) To the nearest tenth, Jay's class standard deviation is 4.5. 	4, 2 or 0 marks	/4
15	Marie-Louise's Z-score is 1.48 . Julius' Z-score is 1.43 . To the nearest hundredth, the difference between their Z-scores is 0.05 .	4, 2 or 0 marks	/4
	Note: Accept also Marie-Louise's Z-score is 1.41. Julius' Z-score is 1.36. To the nearest hundredth, the difference between their Z-scores is 0.05	;.	

Marking scale for developed-response questions

- 1. Unless otherwise indicated, the marking scale included in this document will be used to grade questions 16 to 25, the developed-response questions in the examination. Adherence to the scale will help ensure equity for all students who write the examination.
- 2. Students' work will be analyzed carefully and then evaluated according to the criteria defined herein.
- 3. Students who do not show their work will be given no marks for a correct final answer.

Definition of the Terms Found in the Marking Scale

Appropriate method

A procedure consisting of a series of steps that make it possible to solve the problem.

A student's method is deemed appropriate if the steps presented could lead to the solution.

A method may be deemed appropriate even if the final answer is incorrect. For instance, a student may make one or more mistakes in applying the relevant operations and relations, yet his or her method may still be considered appropriate.

A method may be deemed appropriate if some of the required steps are not fully shown. In this case, the written information is not clear.

Partially appropriate method

A procedure that will not solve the problem, but which shows that the student has a partial understanding of the problem.

A method may still be considered partially appropriate even if the student makes mistakes in applying operations and relations, or if his or her written information is not very clear.

Inappropriate method

A procedure that will not solve the problem and which shows no evidence that the student has any understanding of the problem. Students who do not show their work are deemed to have used an inappropriate method.

Correct application of operations and relations

The student made no mistakes in applying the chosen operations and relations.

Clear written information

The information is complete, legible and presented using correct language.

As a result, the scorer does not need to interpret what the student has done.

To help the scorer, some developed-response questions specify what constitutes a partially appropriate method.



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Part C

Questions 16 to 254 marks eachNo marks are to be given if work is not shown. Examples of correct solutions are given.However, other acceptable solutions are possible.

Example of an appropriate solution

 $\left(m \ \overline{AB}\right)^2 = \left(m \ \overline{AC}\right)\left(m \ \overline{AD}\right)$

Let

16

17

$$x = m \overline{AC}$$

$$(x)(x + 10) = 12^{2}$$

$$x^{2} + 10x = 144$$

$$x^{2} + 10x - 144 = 0$$

$$(x - 8)(x + 18) = 0$$

$$x = 8 \text{ or } x = -18$$



Example of an appropriate solution

Let arc AB = xLet arc CD = 2xInterior angle

$$\frac{2x + x}{2} = 66$$
$$2x + x = 132$$
$$3x = 132$$
$$x = 44$$

Arc CD

$$2x = 2(44)$$
$$= 88^{\circ}$$

Answer: The measure of arc CD is **88**°.

/4

19

Example of an appropriate solution



1 mark for each correctly graphed inequality

Example of an appropriate solution

Initial value (old value) Rule $\rightarrow y = a(1.045)^x$

> Given → $25\ 000 = a(1.045)^{2.5}$ $25\ 000 = a(1.1163)$ $22\ 394.9 \approx a$

Answer: 22 394 people live in Expoville today.

Note: Do not penalize students who have rounded incorrectly.

/4

21

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{(\sin x)(\sin x)}{(\cos x)(\sin x)} + \frac{(\cos x)(\cos x)}{(\cos x)(\sin x)}$$
$$= \frac{\sin^2 x + \cos^2 x}{(\cos x)(\sin x)}$$
$$= \frac{1}{(\cos x)(\sin x)}$$
$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$
$$= \sec x \cdot \csc x$$



Example of an appropriate solution

Centre of circle is a point in square root function (-1, 7)

$$y = a\sqrt{-(x-h)} + k$$
 vertex (8, 5)

Using (-1, 7)

$$7 = a\sqrt{-(-1-8)} + 5$$
$$2 = a\sqrt{9}$$
$$\frac{2}{3} = a$$
$$y = \frac{2}{3}\sqrt{-(x-8)} + 5$$

The equation of the square root function is $y = \frac{2}{3}\sqrt{-(x-8)} + 5$. Answer:

Example of an appropriate solution

/4

Example of an appropriate solution

Absolute Value Function

Vertex

(5, 4)

Zeros

$$0 = -2|x - 5| + 4$$

-4 = -2|x - 5|
2 = |x - 5|
x = 3 or x = 7



Sine function

or

period = 8
amplitude = 4

$$b = \frac{2\pi}{p} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$y = a \sin(b(x - h)) + k$$

$$y = 4 \sin \frac{\pi}{4}(x - 7)$$

$$y = 4 \sin \frac{\pi}{4}(x + 1)$$

Answer: A rule of correspondence is $y = 4 \sin \frac{\pi}{4}(x-7)$ or $y = 4 \sin \frac{\pi}{4}(x+1)$.

Note: Accept any other equivalent rule of correspondence e.g. $f(x) = -4 \sin \frac{\pi}{4}(x-3)$.



/4

Example of an appropriate solution

Parabola

Axis of symmetry

$$h = -\frac{b}{2a}$$
$$= -\frac{2}{2\left(-\frac{1}{5}\right)}$$
$$= 5$$

Maximum Value

$$k = -\frac{1}{5}(5)^{2} + 2(5)$$
$$= -\frac{25}{5} + 10$$
$$= 5$$

Circle centre

(5, 2.5)

Circle radius

$$\frac{5}{2} = 2.5$$

Equation

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-5)^2 + (y-2.5)^2 = 6.25$$

Answer: The equation of the circle is $(x-5)^2 + (y-2.5)^2 = 6.25$.

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Example of an appropriate solution Centre of ellipse (0, 0) Half measure of semi-major axis $\sqrt{25} = 5$ Half measure of semi-minor axis $\sqrt{1} = 1$ Centre of circle B (6, 0) Radius 1 Therefore $(x-6)^2 + y^2 = 1$

Answer: The equation of circle B is $(x-6)^2 + y^2 = 1$.

Example of an appropriate solution

Using trigonometric ratios

m

25

$$\angle POR = PR$$

= $\tan^{-1}\frac{4}{3}$ or $\sin^{-1}\frac{4}{5}$
= 53.13°

Since $m \angle RPQ = \frac{1}{2} \stackrel{\frown}{PR}$ = $\frac{1}{2} (53.13^{\circ}) = 26.56^{\circ}$ = 26.56°

Answer: To the nearest tenth, the degree measure of \angle RPQ is 26.56.

Note: There are various means to arrive at the answer.



Mathematics

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Question Booklet

Secondary 5

September 2005



INSTRUCTIONS

- 1. Write the required information on the cover page of your Answer Booklet.
- 2. Answer all 25 questions in the Answer Booklet.
- 3. You have 3 hours to complete the exam.
- 4. Each question is worth 4 marks.
- 5. You may use a calculator (with or without graphing display), and a memory aid.
- 6. The following materials are allowed: graph paper, ruler, compass, setsquare and protractor.
- 7. The figures in this booklet have NOT been drawn to scale.
- 8. At the end of the exam period, hand in the Question Booklet and Answer Booklet.



2

Part A Questions 1 to 10

In the Answer Booklet, blacken the letter that corresponds to the answer chosen.

Sandra, Omar and Mario are organizing the school fashion show. During their last meeting, the group established the guidelines for selecting the models.

The total number of models must be no more than 38. There must be at least twice as many female models as male models.

Let x: number of male models y: number of female models

Which of the following system of constraints can describe the situation?

A)	$x \ge 0$	C)	$x \ge 0$
	$y \ge 0$		$y \ge 0$
	$x + y \le 38$		$x + y \le 38$
	$y \ge 2x$		$x \ge 2y$
B)	$x \ge 0$	D)	$x \ge 0$
	$y \ge 0$		$y \ge 0$
	$x + y \le 38$		$x + y \le 38$
	$y \le 2x$		$x \le 2y$

Given the function $f(x) = \frac{2x-3}{x-1}$

What are the equations of the asymptotes of this function?

A) x = 1, y = -1B) x = 1, y = 2C) $x = \frac{3}{2}, y = -1$ D) $x = \frac{3}{2}, y = 2$



The function f is defined by the following rule:

$$f(x) = 2\sqrt{x+2} + 4$$

What is the rule of its inverse?

A)
$$f^{-1}(x) = 0.25(x-4)^2 + 2$$
, where $x \ge 4$

- B) $f^{-1}(x) = 0.5(x-4)^2 2$, where $x \ge 4$
- C) $f^{-1}(x) = 0.25(x-4)^2 2$, where $x \ge 4$
- D) $f^{-1}(x) = 0.5(x-4)^2 + 2$, where $x \ge 4$

4

3

Which of the expressions below expresses the following as a single logarithm?

$$\log_3(x+5) + \log_3(x-9) - \log_3(x+3)$$

A)
$$\log_3 (x-7)$$

B) $\log_3 (x+1)$
C) $\log_3 \left(\frac{x^2 - 4x - 45}{x+3}\right)$
D) $\log_3 \left(x^2 - 5x - 42\right)$





7

The Planning Committee for the 2010 Olympic Winter Games in Vancouver, British Columbia has decided that the stage that will be used in the opening ceremonies will be shaped like a parabola. The organizers want the Olympic flame to be placed at the focus of the parabola, whose vertex is centered at the origin.

On the right is the sketch of the stage.



Which of the following could be a rule of correspondence for the stage?

A) $x^2 = 12y$ B) $x^2 = -12y$ C) $y^2 = 12x$ D) $y^2 = -12x$

The graph on the right represents the function:

$$f(x) = -\sqrt{-qx + 12} + p$$



Which of the following is true?

A)
$$p > 0, q > 0$$
C) $p < 0, q < 0$ B) $p < 0, q > 0$ D) $p > 0, q < 0$







In the circle with centre O, on the right: m $\overline{AD} = 10$ cm

 $m \overline{BD} = 2 cm$ $m \overline{CD} = 3 cm$



What is the measure of the diameter of the circle?

- A) 13 cm
- B) 15 cm

In the circle with centre O, on the right:

- Secant EF intersects the circle at C
- Secant GF intersects the circle at D
- $m \angle EFG = 15^{\circ}$
- m $\overline{\text{CD}} = 50^{\circ}$



17 cm

30 cm

C)

D)

What is the measure of arc EG?

- A) 100° C) 30°
- B) 80° D) 20°



Part B Questions 11 to 15

Write your answer in the space provided in the answer booklet. Show your work, where required.

A garden is in the shape of a right triangle, ABC, as shown below. Angle A measures 90°.

- Segment AD is drawn perpendicular to segment BC
- Segment CD = 5 m
- Segment BD = 18 m

Triangle ACD is to be planted with roses and triangle ABD is to be covered with grass.



To the nearest m², what area of the garden will be covered in grass?

12 What are the *x*-intercepts of the following function?

$$f(x) = -4|x-5| + 10$$



The length of major arc QP in the unit circle below is $\frac{9\pi}{5}$ units.



To the nearest hundredth, what are the coordinates of point P on the unit circle?

14 Mrs. Lenoir is missing some information for two of her students. She has given you the task of finding the missing data. The two students are not in the same class.

Student	Math Mark	Class Mean	Class Standard Deviation	Z-score
Kayla	80	а	5.5	1.82
Jay	90	95	b	-1.11

a) What is Kayla's class mean?

b) What is Jay's class standard deviation, to the nearest tenth?

Mr. Huerta noted that Marie-Louise and Julius were the only two students who received a mark of 98 on their math tests. He wants you to compare their Z-scores.

Marie-Louise's class 50, 53, 62, 67, 69, 78, 81, 84, 98, 99

Julius' class

15

45, 58, 63, 74, 77, 78, 83, 88, 93, 98

To the nearest hundredth, what is the difference between their Z-scores?



Part C Questions 16 to 25

- Show all your **work** as well as your **answer** in the answer booklet. The work shown is taken into consideration when marks are awarded.
- Your written information must be legible, complete, and clearly stated in correct language so the marker understands exactly what you have done.

Even if your answer is correct, no marks will be given unless acceptable work is shown.

A circle with centre O has a tangent AB and a secant ACD, as shown in the diagram. The secant passes through the centre of the circle whose radius is 5 units. Segment AB measures 12 units.



What is the length of segment AC, in units?

In the circle below, secants AC and BD intersect at point E. Angle AEB measures 66°. The measure of arc AB is one half the measure of arc CD.



What is the measure of arc CD?

18

Mr. Nigel and Ms. Williams are organizing the 2005 European Trip for their school. The school will allow no more than fifty students to go on the trip. From past experience, the teachers know that the number of Secondary 5 students who will go on the trip is at least twice the number of Secondary 4 students. Also, Mr. Nigel and Ms. Williams have asked that at least 10 Secondary 4 students be given the opportunity to go on the trip and that no more than 35 Secondary 5 students be allowed to go.

The following system of constraints describes this situation.

Let x = Number of Secondary 5 students y = Number of Secondary 4 students

 $x + y \le 50$ $x \ge 2y$ $y \ge 10$ $x \le 35$

Draw the polygon of constraints that corresponds to this situation.



19 The town of Expoville is growing at a rate of 4.5 % per year. At a recent press conference, the mayor announced that the population of the town will be 25 000 in two and a half years.

How many people live in Expoville today?

Prove the following trigonometric identity:

 $\tan x + \cot x \equiv \sec x \bullet \csc x$

21 Nathalie is designing a logo for a small business. The owner of a fruit store has asked her to design a cherry logo. Here is her design so far:



Nathalie has asked for your help. She wants you to find the equation of the square root function that represents the stem of the cherry. She has given you the following information:

- Equation of the circle: $(x + 1)^2 + (y 7)^2 = 4$
- Vertex of the square root function: V(8, 5)

What is the equation of the square root function that represents the stem of the cherry?



An absolute value function and a sinusoidal function are drawn on the same Cartesian plane, as shown in the diagram below, in which the sinusoidal axis is the x-axis. The rule of correspondence for the absolute value function is f(x) = -2|x - 5| + 4. The two functions intersect at the points C and E and have the same maximum heights.



What is a rule of correspondence for the sinusoidal function?

23

A circle is tangent to a parabola at its vertex. The equation of the parabola is

$$y = -\frac{1}{5}x^2 + 2x$$

The circle is also tangent to the *x*-axis.



What is the equation of the circle?



Two identical crop circles and an ellipse, depicted below, were recently discovered in a farm in St. Anicet. A UFO Study group in Montreal is studying the crop design. So far, the scientists have determined the equation of the ellipse, but they are still trying to find the equations of the circles.

The equation of the ellipse is

$$\frac{x^2}{25} + \frac{y^2}{1} = 1.$$

The radius of the circle is the same as the minimum distance between the centre of the ellipse and a point on the ellipse.



What is the equation of circle B?

25 The equation of the circle with centre O is $x^2 + y^2 = 25$.

The circle intersects the *x*-axis at the points T and R.

A tangent drawn at point P(3, -4) intersects the x-axis at point Q.



To the nearest tenth, what is the degree measure of angle RPQ?



		1		Mathematics
FOR TEACHER US	E ONLY			
Part A	_ /40	Compreh Numb	ensive Exam	
Part B	_ /24			
	10.1	Answer	Booklet	
Part C	/36	Secon	dary 5	
Total	/100	Septem	ber 2005	
ſ		Student	's Name	
		51000011		
		Group	Date	



	Siz		Answer Book	let		Page 1
	Part A	Questions 1 to 10 Blacken the letter that Each question is wort	t corresponds h 4 marks.	to the ans	wer chosen.	
1	[A] [B] [C] [D]	6	[A] [B]	[C] [D]	
2	[A] [B] [C] [D]	7 [[A] [B]	[C] [D]	
3	[A] [B] [C] [D]	8 [[A] [B]	[C] [D]	
4	[A] [B] [C	[D]	9 [[A] [B]	[C] [D]	
5	[A] [B] [C	[D]	10	[A] [B]	[C] [D]	
	Part B	Questions 11 to 15 Write your answer in	the space pro	ovided.		
11	To the nearest	t^2 m ² , the garden area cover	red in grass is		-	4 0
12	The <i>x</i> –interce	pts are and				4 2 0
13	To the nearest	hundredth, the coordinate	es of point P a	are		4 2 0
	(,).				
14	a) Kayla's	s class mean is				4 2 0
	b) To the	nearest tenth, Jay's class s	standard devi	ation is	·	
15	Marie-Louise'	s Z-score is				4 2 0
	Julius' Z-score	e is				
	To the nearest	hundredth, the difference	between thei	r Z-scores	is	

Part C Questions 16 to 25

- Show all your **work** as well as your **answer**. The work shown is taken into consideration when marks are awarded.
- Your written information must be legible, complete, and clearly stated in correct language so the marker understands exactly what you have done.

Even if your answer is correct, no marks will be given unless acceptable work is shown.













4	3	2	1	0

Show all your work.

Answer: ______ people live in Expoville today.



4	3	2	1	0

$\tan x + \cot x \equiv \sec x \bullet \csc x$	
$\lim x + \cot x = 500 x + 00500 x$	





	4 3 2 1 0
Show all your work. Equation of parabola: $y = -\frac{1}{5}x^2 + 2x$	



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