GRICS

568-526

Mathematics

Comprehensive Exam
Number 54

GUIDE

Secondary 5

September, 2004

Youth Sector General Education

1. GENERAL INFORMATION

1.1	Program	Mathematics, Secondary 5 (568-526)
1.2	Origin	Mathematics Coordinating Committee Examination, 2004 Computerization by Marie-Josée Mickel, BIM, Société GRICS
1.3	Time allotted	3 hours
1.4	Number of questions	25 questions distributed as follows 10 multiple-choice questions 5 short-answer questions 10 developed-response questions
1.5	Authorized material	• Ruler, compass, set square, protractor • Graph paper • Scientific calculator with or without a graphic display The calculator must be portable and designed primarily to perform mathematical calculations. Computers and calculators with a QWERTY keyboard, symbol manipulation capabilities or an electronic date book are not permitted. User guides, memory expansion features or any other calculator accessories are not permitted during the examination. Students may not share their calculators with other students. Communication links between calculators are also forbidden during the examination. • Memory aid The memory aid is **one** letter-size sheet of paper (8½ × 11") on which a student will have recorded information of his or her choice. Students are encouraged to work on and revise their memory aid throughout the year. Both sides of the sheet may be used. Any mechanical reproduction of this memory aid is forbidden. Students may not share their memory aids with any other students.

2. DESCRIPTION OF THE EXAM

The chart below shows the distribution of the items taking into account the relative importance given to the different components of the program.

Exam Specifications

Themes / Skills	Algebra 68%	Geometry 20%	Statistics 12%
Mastery of Concepts 28%	1, 2, 3, 4, 5	11	8
Mastery of Application 40%	6, 7, 13, 14, 15, 16	10, 12, 22	9
Mastery of Problem Solving Techniques 32%	17, 18, 19, 20, 23, 24	21	25

Note The numbers in the centre of each box represent the question numbers in the examination.

Item Specifications

Question	Item	Objective	S	T	D
Part A					
1	2026	ALG.01.01	C	M	E
2	2027	ALG.01.03	C	M	M
3	2028	ALG.02.04	C	M	M
4	2029	ALG.02.04	C	M	E
5	2030	ALG.02.01	C	M	M
6	2031	ALG.02.04	A	M	M
7	2032	ALG.02.04	A	M	M
8	2033	STA.01.06	C	M	E
9	2034	STA.01.02	A	M	M
10	2035	GEO.01.02	A	M	E
Part B					
11	2036	GEO.01.03	C	C	E
12	2037	GEO.01.02	A	C	M
13	2038	ALG.04.02	A	C	M
14	2039	ALG.03.03	A	C	M
15	2040	ALG.03.01	A	C	M
16	2041	ALG.03.04	A	C	M
Part C					
17	2042	ALG.01.05	P	E	M
18	2043	ALG.02.03	P	E	M
19	2044	ALG.02	P	E	D
20	2045	ALG.02	P	E	D
21	2046	GEO.01	P	E	M
22	2047	GEO.01	A	E	M
23	2048	STA.01	P	E	M
24	2049	ALG.04	P	E	D
25	2050	ALG.04	P	E	D

Legend

S: Skill
- A: Application
- C: Concept
- P: Problem solving

T: Type of items
- C: Short-constructed answer
- E: Extended answer (developed response)
- M: Multiple choice

D: Level of Difficulty
- E: Easy
- M: Medium
- D: Difficult

3. INSTRUCTIONS FOR TEACHERS

- Ensure that each student has all the material needed.
- Hand out the Question Booklets and read the instructions aloud to the students.
- Collect all booklets at the end of the examination.

4. CORRECTION KEY

Part A

Questions 1 to 10 ***4 marks or 0 marks***

1	D	**6**	D
2	B	**7**	C
3	A	**8**	B
4	D	**9**	D
5	A	**10**	C

Part B

Questions 11 to 16 ***4 marks each***

11 Example of an appropriate solution /4

2. In a circle, an inscribed angle measures one half of its intercepted arc.

3. In a circle, the measure of the central angle is equal to the degree measure of its intercepted arc.

2 marks for each justification

12 The degree measure of arc $\overset{\frown}{EF}$ is 24°. *4 marks or 0 marks* /4

13 $\frac{x^2}{56.25} + \frac{y^2}{36} = 1$ or equivalent $\left(\text{e.g. } \frac{4x^2}{225} + \frac{y^2}{36} = 1 \text{ or } \frac{x^2}{(7.5)^2} + \frac{y^2}{6^2} = 1\right)$. *4 marks or 0 marks* /4

14 $x = 4$ *4 marks*
$x = 4$ or $x = -9$ *2 marks* /4

15 Example of an appropriate solution /4

$$
\begin{aligned}
P(x) &= 25|x - 150| - 1200 \\
0 &= 25|x - 150| - 1200 \\
1200 &= 25|x - 150| \\
48 &= |x - 150| \\
x - 150 = 48 \quad &\text{or} \quad x - 150 = -48 \\
x = 198 \quad &\quad\;\; x = 102
\end{aligned}
$$

$198 - 102 + 1 = 97$

The company had either no profit or a loss for **97** days.

Note: Accept also $198 - 102 = 96 \quad \therefore$ 96 days.

4 marks for an appropriate method and the correct answer.
3 marks for an appropriate method and finding both zeros of the function.
2 marks for an appropriate method and finding one zero of the function.

16 Example of an appropriate solution /4

$$\csc^2 x + \sec^2 x = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$ *1 mark*

$$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$$ *2 marks*

$$= \frac{1}{\sin^2 x \cos^2 x}$$ *1 mark*

Marking scale for developed-response questions

1. Unless otherwise indicated, the marking scale included in this document will be used to grade questions 17 to 25, the developed-response questions in the examination. Adherence to the scale will help ensure equity for all students who write the examination.

2. Students' work will be analyzed carefully and then evaluated according to the criteria defined herein.

3. **Students who do not show their work will be given no marks for a correct final answer**.

Definition of the Terms Found in the Marking Scale

Appropriate method

A procedure consisting of a series of steps that make it possible to solve the problem.

A student's method is deemed appropriate if the steps presented could lead to the solution.

A method may be deemed appropriate even if the final answer is incorrect. For instance, a student may make one or more mistakes in applying the relevant operations and relations, yet his or her method may still be considered appropriate.

A method may be deemed appropriate if some of the required steps are not fully shown. In this case, the written information is not clear.

Partially appropriate method

A procedure that will not solve the problem, but which shows that the student has a partial understanding of the problem.

A method may still be considered partially appropriate even if the student makes mistakes in applying operations and relations, or if his or her written information is not very clear.

Inappropriate method

A procedure that will not solve the problem and which shows no evidence that the student has any understanding of the problem. Students who do not show their work are deemed to have used an inappropriate method.

Correct application of operations and relations

The student made no mistakes in applying the chosen operations and relations.

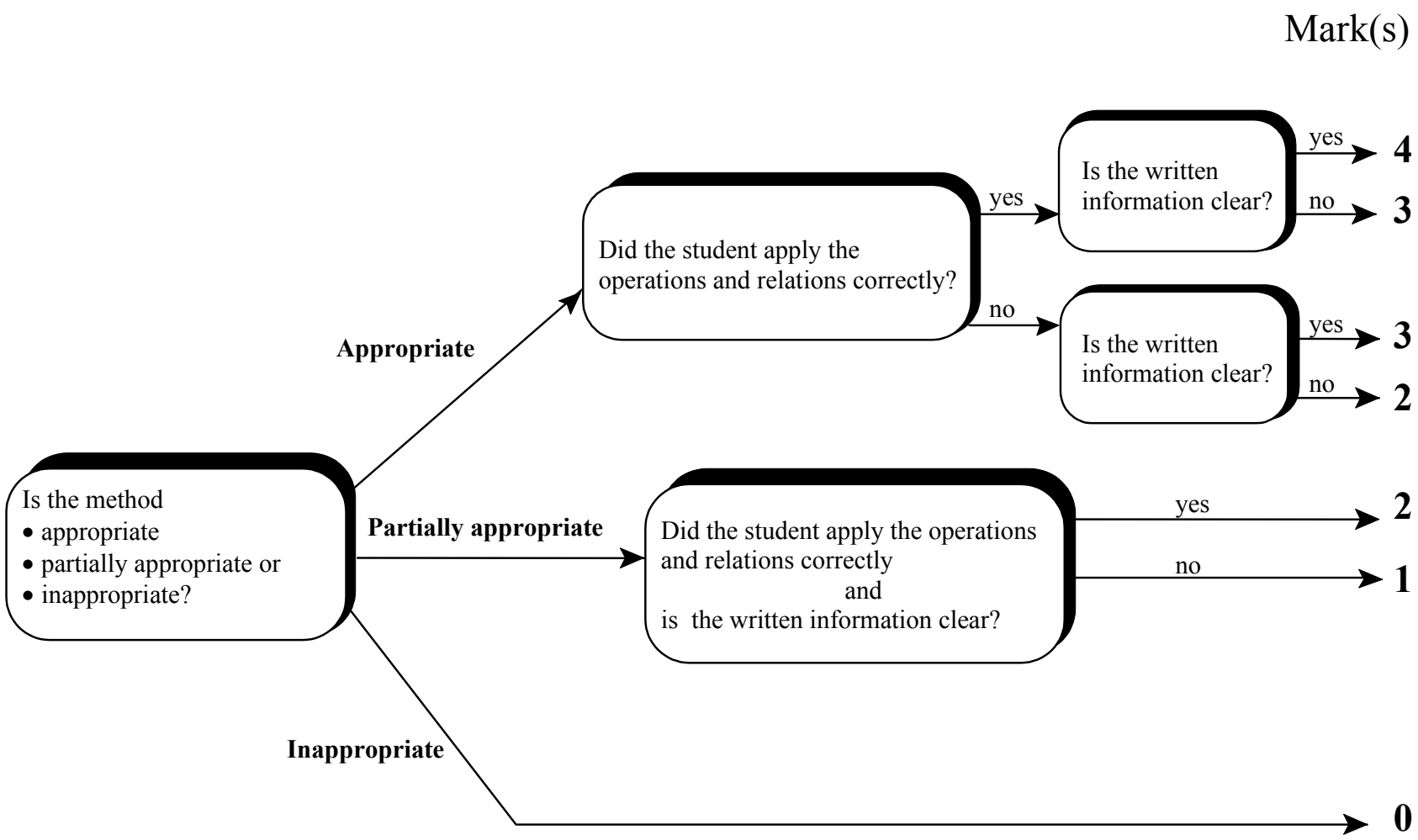
Clear written information

The information is complete, legible and presented using correct language.

As a result, the scorer does not need to interpret what the student has done.

To help the scorer, some developed-response questions specify what constitutes a partially appropriate method.

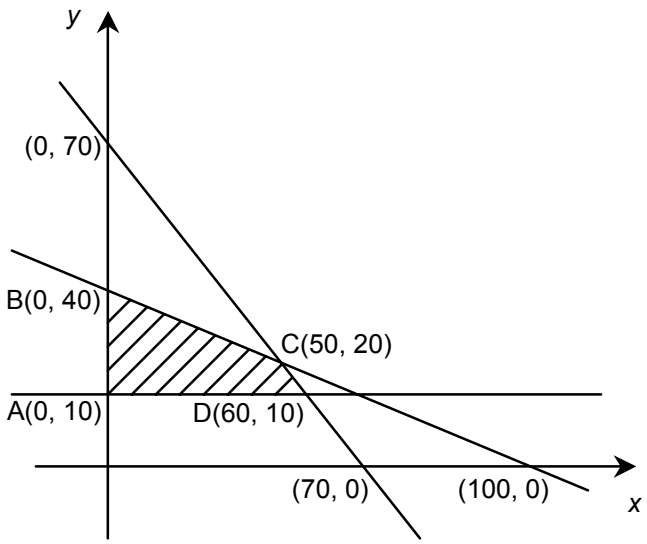
MARKING SCALE



Part C

Questions 17 to 25 ***4 marks each***
No marks are to be given if work is not shown. Examples of correct solutions are given. However, other acceptable solutions are possible.

17 Example of an appropriate solution /4

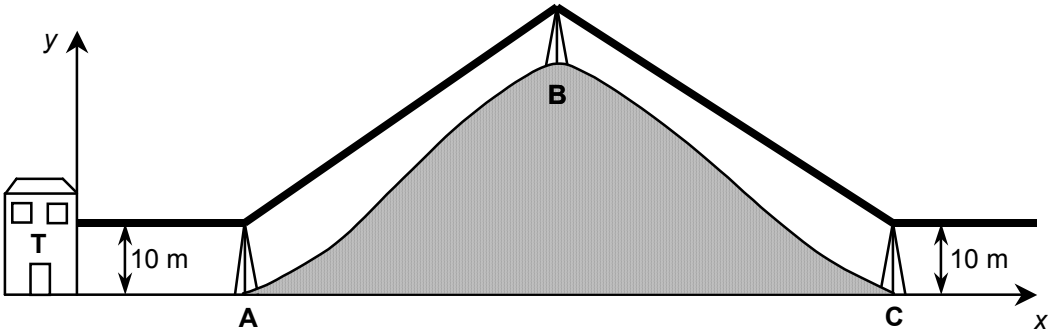


Vertex	Weekly Revenue = 400x + 420y
A(0, 10)	$4200
B(0, 40)	$16 800
C(50, 20)	**$28 400**
D(60, 10)	$28 200

Answer: The company must sell of **50** gas and **20** electric mowers to maximize its revenue.

Note: Students who use an appropriate method to graph the polygon of constraints have shown they have a partial understanding of the problem.

18 Example of an appropriate solution /4



Vertex (150, 40)

$$10 = -\frac{1}{2}|x - 150| + 40$$

$$-30 = -\frac{1}{2}|x - 150|$$

$$60 = |x - 150|$$

$60 = x - 150$ $\qquad$ $-60 = x - 150$

$x = 210$ $\qquad$ $x = 90$

$P_1(90, 10)$ $\qquad$ $P_2(210, 10)$

$$\overline{P_1V} = \sqrt{(150 - 90)^2 + (40 - 10)^2}$$

$$\overline{P_1V} = 67.08$$

$$\overline{P_2V} = 67.08$$

Answer: To the nearest tenth of a metre, the total length of the power line connecting the three towers is **134.2** m.

Note: Allot 3 marks to students who appropriately and correctly solved the problem using $y = 0$ and obtained the answer 178.9 cm.

Students who use an appropriate method to determine the coordinates of the vertex and the coordinates of points P_1 and P_2 have shown that they have a partial understanding of the problem.

19 Example of an appropriate solution /4

Mario's investment

$$A = P(1 + r)^t$$
$$10\,000 = 5000(1 + 0.05)^t$$
$$\frac{10\,000}{5000} = (1.05)^t$$
$$2 = 1.05^t$$
$$\log 2 = \log 1.05^t$$
$$\frac{\log 2}{\log 1.05} = t$$
$$14.2 = t$$

Tina's investment

$A = P(1 + r)^t$
$A = 3000(1 + 0.06)^{14.2}$
$A = 3000(1.06)^{14.2}$
$A = 6862.22$

Answer: By the time that Mario has doubled his investment, Tina will have accumulated $**6862.22**.

Note: Do not penalize students who did not round *t* or rounded inappropriately, obtaining answers ranging from 6862 to 6865.

Students who used an appropriate method to determine the value of *t* have shown they have a partial understanding of the problem.

20 Example of an appropriate solution /4

Trigonometric Function

$$g(x) = 4\cos\left(\frac{\pi x}{3}\right)$$

Amplitude = 4

$$b = \frac{\pi}{3}$$

$$P = \frac{2\pi}{b}$$

$$= \frac{2\pi}{\frac{\pi}{3}}$$

$$= 6$$

=> x-intercepts are

$$\frac{1}{4}(6) = 1.5$$

$$\frac{3}{4}(6) = 4.5$$

etc.

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$a = 4.5$, $b = 4$

Answer: $\boldsymbol{\frac{x^2}{4.5^2} - \frac{y^2}{4^2} = -1}$ or equivalent

Note: Students who use an appropriate method in order to find the value of the zero which corresponds to the value of parameter a in the equation of the hyperbola have shown a partial understanding of the problem.

21 Example of an appropriate solution /4

The length of a leg of a right triangle is the geometric mean (mean proportional) between the length of the hypotenuse and the length of the adjacent segment of the hypotenuse formed by the altitude to the hypotenuse.

$$\left(m\,\overline{BC}\right)^2 = \left(m\,\overline{BF}\right)\left(m\,\overline{BD}\right)$$
$$\left(m\,\overline{BC}\right)^2 = (7)(26)$$
$$\left(m\,\overline{BC}\right)^2 = 182$$
$$m\,\overline{BC} = 13.49$$

$$\left(m\,\overline{CD}\right)^2 = \left(m\,\overline{DF}\right)\left(m\,\overline{DB}\right)$$
$$\left(m\,\overline{CD}\right)^2 = (19)(26)$$
$$\left(m\,\overline{CD}\right)^2 = 494$$
$$m\,\overline{CD} = 22.23$$

13.49 + 22.23 = 35.72 metres

Answer: Rounded to the nearest tenth of a metre, Mary swims a total of **35.7** metres.

Note: Do not penalize students who do not round to the nearest tenth of a metre.

Students who use an appropriate method in order to find $\overline{BC}$ or $\overline{CD}$ have shown they have a partial understanding of the problem.

22 Example of an appropriate solution /4

Statement	Justification
$\frac{m\,\overset{\frown}{AB} - m\,\overset{\frown}{DE}}{2} = m\angle ACB$	An angle whose vertex is outside the circle measures half the difference of its intercepted arcs.
$\frac{150 - x}{2} = 55$	
$m\,\overline{DE} = 40°$	
$m\angle DFE = 20°$	An inscribed angle is equal to half its intercepted arc.
$m\angle FDO = 20°$	$\overline{OD}$ and $\overline{OF}$ are equal radii and the angles adjacent to the congruent sides of an isosceles triangle are equal.

Answer: The measure of $\angle$ FDO is **20°**.

Note: Students who use an appropriate method to determine $m\,\overset{\frown}{DE}$ with appropriate justification have shown they have a partial understanding of the problem.

Students who use an appropriate method to determine the measure of $\angle$ FDO but have not justified their work have shown they have a partial understanding of the problem.

23 Example of an appropriate solution /4

Note: Accept answers calculated using either σ or s.

Original set

$\bar{x} = 71.35$ or $\bar{x} = 71.35$

$\sigma = 19.05$ $s = 19.55$

$$z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{s}$$

$$= \frac{100 - 71.35}{19.55} \qquad = \frac{100 - 71.35}{19.05}$$

$$= 1.5 \qquad = 1.47$$

Revised set

$n = 19$ $n = 19$

$\bar{x} = 74.58$ $\bar{x} = 74.58$

$\sigma = 13.17$ $s = 13.53$

$$z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{s}$$

$$1.5 = \frac{x - 74.58}{13.17} \qquad 1.47 = \frac{x - 74.58}{13.53}$$

$$x = 94.3 \qquad x = 94.5$$

Answer: The new minimum mark required to win the prize was **94**%.

Note: Do not penalize students who have not rounded their final answer or who have rounded incorrectly.

Students who use an appropriate method to find the required Z-score (1.5 or 1.47) have shown they have a partial understanding of the problem.

24 Examples of an appropriate solution /4

Centre of the circle is (8, 6)

Equation of one of the tangents

Slope of radius to point of tangency (9, 4)

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 6}{9 - 8} = -2$$

Slope of tangent at (9, 4) is therefore $\frac{1}{2}$

Thus, the equation of the tangent at (9, 4) is $y = 0.5x - 0.5$

Coordinates of the corner (i.e. the vertex of the absolute value function)

The x-coordinates of the corner point is 8, (symmetry).

$y = 0.5(8) - 0.5$

$= 3.5$

Corner point: (8, 3.5)

Minimal distance between the disc and the corner

The distance from the centre of the circle to the lowest point on the circle is the radius, which equals $\sqrt{5}$ cm.

The distance from the centre of the circle (8, 6) to the corner (8, 3.5) is

$6 - 3.5 = 2.5$ cm.

Therefore the minimal distance separating the disc and the corner is $2.5 - \sqrt{5}$.

Answer: To the nearest tenth of a centimetre, the minimal distance separating the disc and the corner of the room is **0.3** cm.

Note: Students who use an appropriate method to determine the equation of one of the walls (i.e. tangents), or the equation of an equivalent absolute value function, have shown they have a partial understanding of the problem.

25 Example of an appropriate solution /4

Parabola

$x^2 = -20y$

$4c = -20$

$c = -5$

The directrix has equation

$y = 5$

Absolute value function $y = a|x - h| + k$

$(h, k) = (0, 10) \Rightarrow y = a|x| + 10$

Point $(6, 5) \Rightarrow 5 = a|6| + 10$

$a = \frac{-5}{6}$

Base

$x^2 = -20y$

$y = -12 \Rightarrow x^2 = -20(-12)$

$x^2 = 240$

$x = \pm \sqrt{240}$

$x = \pm 15.5$ (rounded to nearest tenth)

$y = \frac{-5}{6}|x| + 10$

$y = -12 \Rightarrow -12 = \frac{-5}{6}|x| + 10$

$|x| = 26.4$

$x = \pm 26.4$

$m\ \overline{AB} = 26.4 - 15.5 = 10.9$ m

Answer: To the nearest tenth of a metre, the width of the base $\overline{AB}$ is **10.9** m.

Note: Students who use an appropriate method to find an equation for the absolute value function (or a linear function corresponding to one of the branches of the absolute value function) have shown they have a partial understanding of the problem.

GRICS

568-526

Mathematics

Comprehensive Exam
Number 54

Question Booklet

Secondary 5

September 2004

Youth Sector General Education

INSTRUCTIONS

1. Write the required information on the cover page of your Answer Booklet.

2. Answer all 25 questions in the Answer Booklet.

3. You have 3 hours to complete the exam.

4. Each question is worth 4 marks.

5. You may use a calculator (with or without graphing display), and a memory aid.

6. The following materials are allowed: graph paper, ruler, compass, setsquare and protractor.

7. The figures in this booklet have NOT been drawn to scale.

8. At the end of the exam period, hand in the Question Booklet and Answer Booklet.

Part A **Questions 1 to 10**

In the Answer Booklet, blacken the letter that corresponds to the answer chosen.

1 A school student council has raised money to hold a Ski Day this year. Students will be able to spend the day skiing or skating. The amount of money raised will cover the expenses for no more than one hundred students. Since skiing is more expensive, there must be more students skating than skiing.

Let x: number of students who ski
y: number of students who skate

Which of the following systems of constraints could describe this situation?

A)
$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 100$$
$$x > y$$

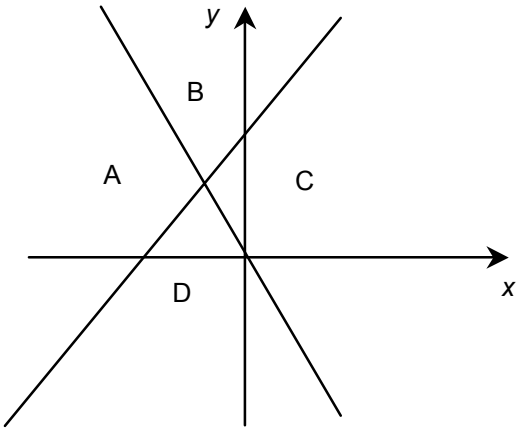
B)
$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 100$$
$$x > y$$

C)
$$x \geq 0$$
$$y \geq 0$$
$$x + y \geq 100$$
$$y > x$$

D)
$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 100$$
$$y > x$$

2 **Which lettered region of the Cartesian plane below represents the solution set to the following system of inequalities?**

$$x - y \leq -6$$
$$2x + y \geq 0$$



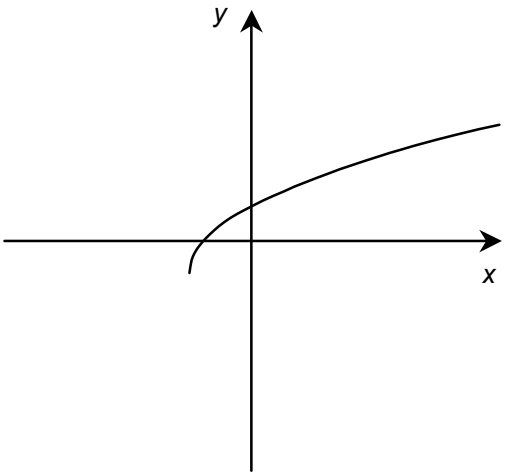
A) Region A

B) Region B

C) Region C

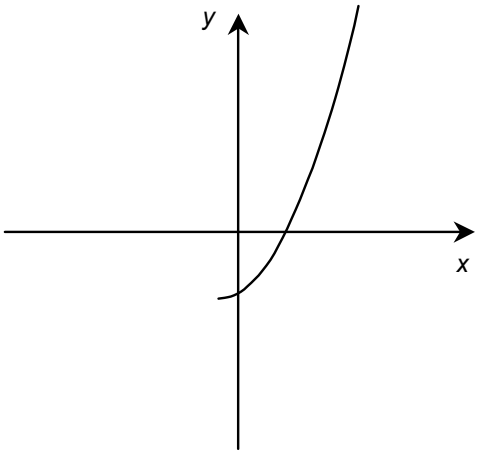
D) Region D

3 Consider the graph of the function shown in the Cartesian plane on the right.

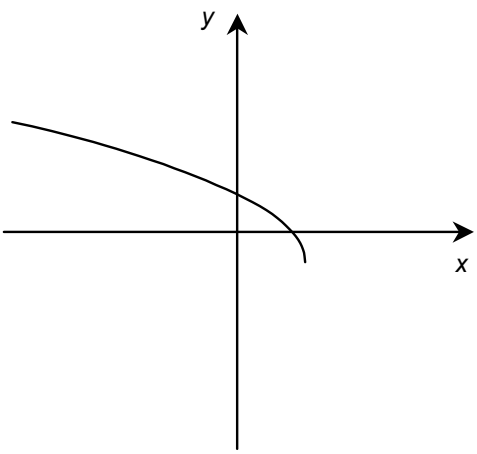


Which of the following represents the graph of the inverse of the function?

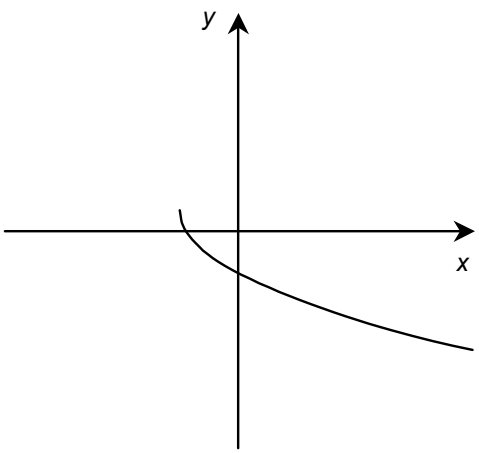
A)



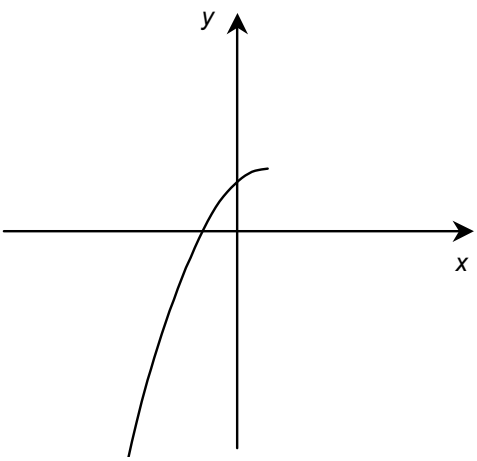
C)



B)



D)



4 Consider the rational function *f* of the form

$$f(x) = \frac{ax + b}{cx + d}, \qquad \text{and } a, b, c, d \neq 0.$$

Which of the following represents the equation of the vertical asymptote?

A) $x = \frac{a}{c}$

B) $x = \frac{b}{d}$

C) $x = \frac{-b}{a}$

D) $x = \frac{-d}{c}$

5 The graph of the basic square root function $f(x) = \sqrt{x}$ is reflected in the *y*-axis and then translated one unit to the right.

What is the rule of the resulting function *g*(*x*)?

A) $g(x) = \sqrt{-(x-1)}$

B) $g(x) = -\sqrt{x} + 1$

C) $g(x) = -\sqrt{x-1}$

D) $g(x) = \sqrt{-x} + 1$

6 Student council members want to organize a talent show at their school to raise money for a charity. The school's auditorium can accommodate a maximum of 200 people. Tickets will cost $15 each. The council has established that the average profit per person in attendance can be calculated using the following function, where P(x) represents the average profit per person if x people buy tickets.

$$\mathrm{P}(x) = \frac{15x - 2250}{x}$$

Given the size of their auditorium, which one of the following statements is TRUE?

A) The maximum achievable profit per person is $15.

B) The student council will be able to donate $3000 to the charity.

C) The student council will not be able to raise money for the charity.

D) More than 150 people must attend in order to raise money for the charity.

7 The intensity, $f(x)$, in lumens emitted by a light source varies periodically over time (x) according to the rule

$$f(x) = -4 \sin \frac{\pi}{6}(x - 3) + 8$$

Over which of the following intervals of the domain is the intensity strictly increasing?

A) [-4, 4]

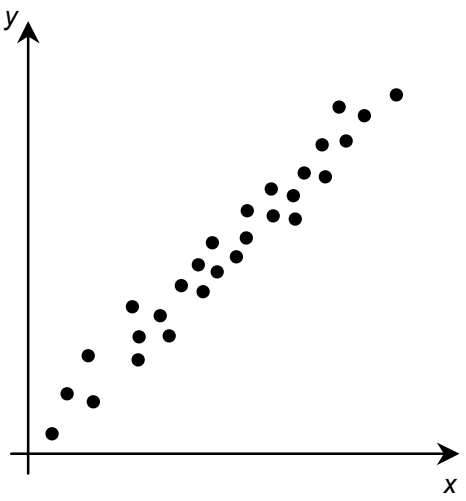
B) [4, 12]

C) [6, 12]

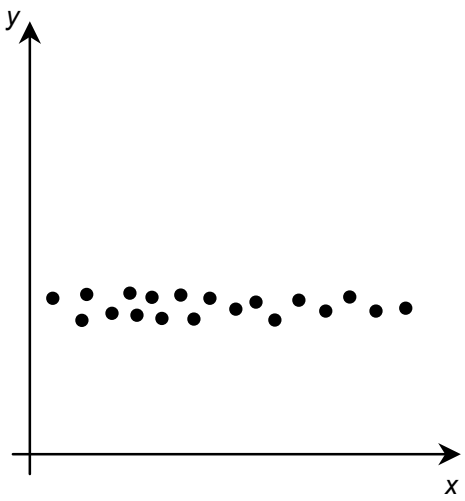
D) [12, 18]

8 **Which of the distributions below has a correlation coefficient closest to 1?**

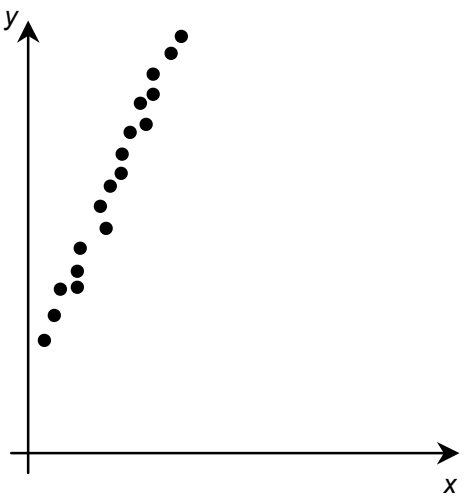
A)



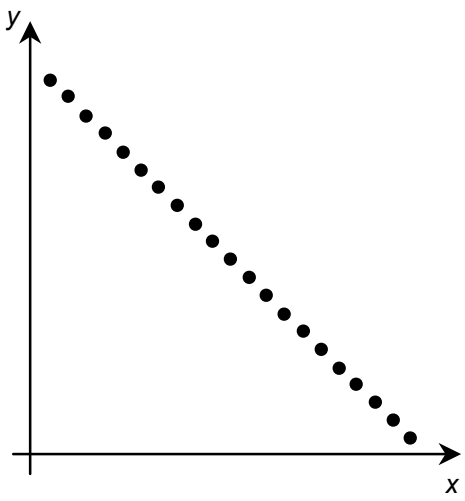
C)



B)



D)



9 The following table represents the marks obtained by students in two different classes.

Class A	53, 56, 66, 68, 69, 71, 72, 73, 77, 80, 82, 85, 87, 91, 95
Class B	61, 66, 69, 72, 75, 77, 78, 79, 81, 82, 85, 88, 92, 96, 99

The standard deviation in class A is 12.01, and in class B it is 10.81.
Johnny received a mark of 87% in class A, and Betty received a mark of 92% in class B.

Which one of the following statements is TRUE?

A) Johnny's and Betty's Z-scores are equal.

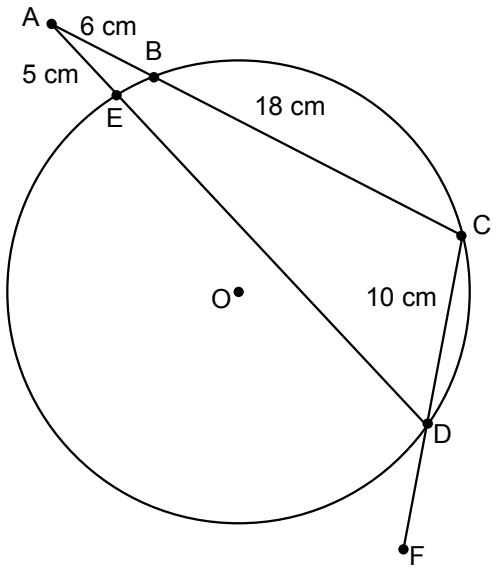
B) The range in class A is less than in class B.

C) The class average is higher in class A than in class B.

D) Relative to their classes, Betty's mark is better than Johnny's.

10 In the diagram on the right:

- Point O is the centre of the circle
- Secant $\overline{AC}$ intersects the circle at point B
- Secant $\overline{AD}$ intersects the circle at point E
- Secant $\overline{CF}$ intersects the circle at point D



m $\overline{AB}$ = 6 cm
m $\overline{BC}$ = 18 cm
m $\overline{CD}$ = 10 cm
m $\overline{AE}$ = 5 cm

What is the perimeter of triangle ACD?

A) 60.0 cm

B) 60.6 cm

C) 62.8 cm

D) 67.8 cm

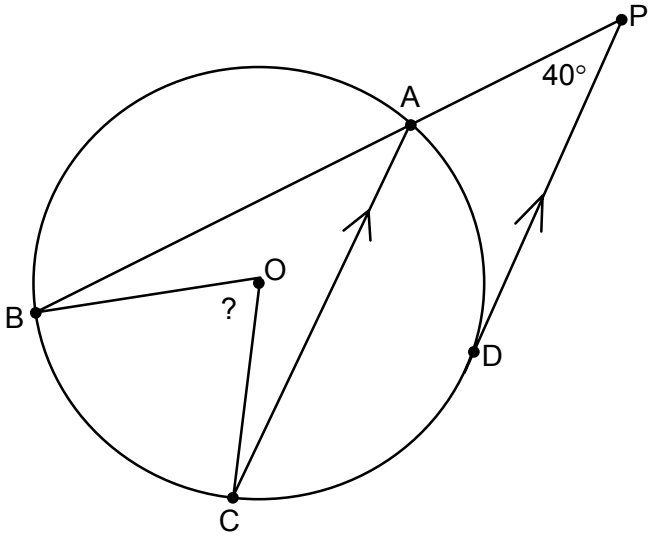
Part B **Questions 11 to 16**

Write your answer in the space provided in the answer booklet.
Show your work, where required.

11 In the circle with centre O,

$\overline{PD}$ is tangent to the circle
$\overline{PD}$ is parallel to $\overline{AC}$
m $\angle$ BPD = 40°

What is the measure of $\angle$ BOC?



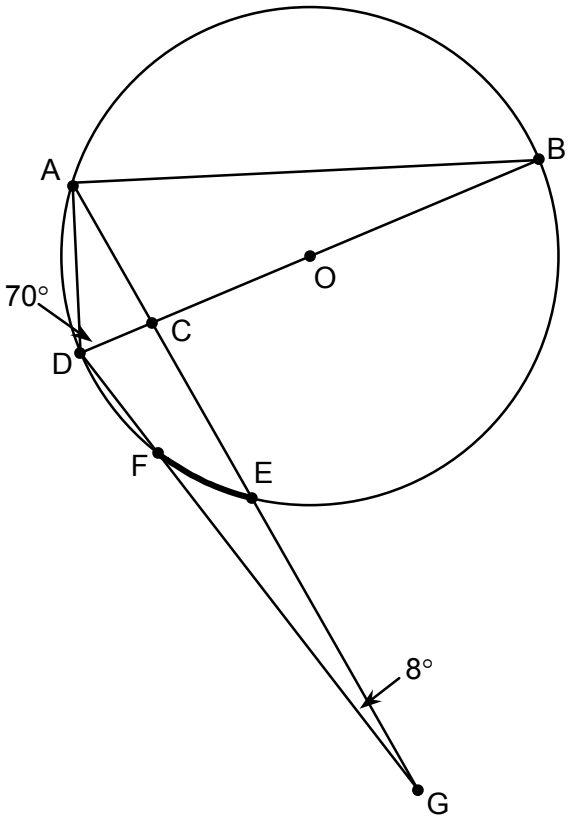
Write the missing justifications in the solution of this problem.

Statement	Justification
1. m $\angle$ BAC = 40°	If a transversal intersects two parallel lines, then the corresponding angles are isometric.
2. m $\overset{\frown}{BC}$ = 80°	?
3. m $\angle$ BOC = 80°	?

12 In the diagram on the right:

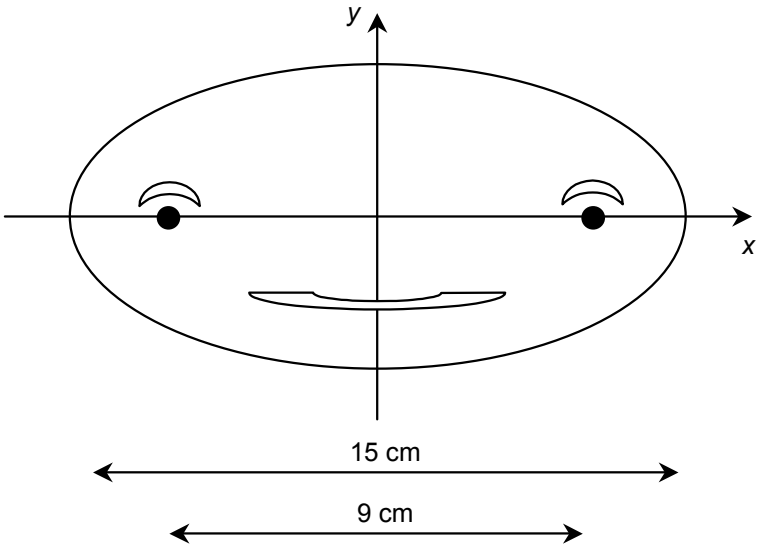
- O is the centre of the circle
- Secant $\overline{GA}$ intersects the circle at E
- Secant $\overline{GD}$ intersects the circle at F

m $\angle$ ADC = 70°
m $\angle$ AGD = 8°



What is the degree measure of arc EF?

13 A company's design department came up with a smiling face logo for its advertising campaign. To make it easier to work with the design on a computer, the face was drawn in the shape of an ellipse with its centre at the origin of a Cartesian plane, as shown in the diagram below. The axes are scaled in centimetres.



The face was drawn 15 cm wide. The centres of the eyes were located at the foci of the ellipse and were 9 cm apart.

What equation represents the ellipse?

14 **Find the solution or solutions of the logarithmic equation shown below.**

$$\log_6(x) + \log_6(x + 5) = 2$$

15 The daily profit P(x) for a small company with respect to the number of days x that have passed since the beginning of the year can be calculated using the following absolute value function:

$$P(x) = 25|x - 150| - 1200$$

For how many days during the year did the company have either no profit or a loss?

16 **Prove the following trigonometric identity:**

$$\csc^2 x + \sec^2 x = \frac{1}{\sin^2 x \cos^2 x}$$

Part C **Questions 17 to 25**

- Show all your **work** as well as your **answer**. The work shown is taken into consideration when marks are awarded.
- Your written information must be legible, complete, and clearly stated in correct language so the marker understands exactly what you have done.

Even if your answer is correct, no marks will be given unless acceptable work is shown.

17 A company makes two types of lawnmowers: gas and electric powered. The following table gives the necessary time, in hours, for assembly and inspection for one of each type of lawnmower.

Work	Gas	Electric
Assembly	2	5
Inspection	1	1

The employees working on assembly can work a maximum of 200 hours per week, and those working on inspection can work a maximum of 70 hours per week. There must be at least 10 electric mowers produced weekly. This situation can be represented by the following system of inequalities:

Let x: number of gas mowers
y: number of electric mowers

$$x \geq 0,\ y \geq 0$$
$$2x + 5y \leq 200$$
$$x + y \leq 70$$
$$y \geq 10$$

Gas powered and electric powered lawnmowers are sold for \$400 and \$420 respectively.

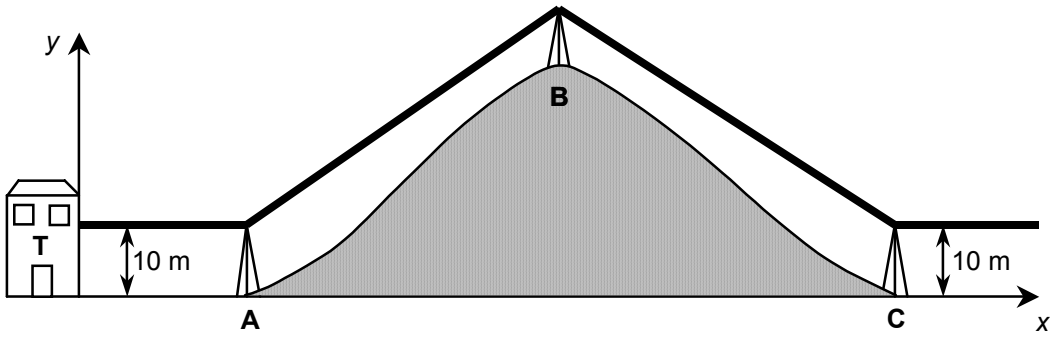
How many lawnmowers of each type must this company sell per week to maximize its revenue?

18 A power line leaves a transformer station T and passes over a hill. Three towers, A, B, and C support the power line. Tower B stands on the summit of the hill; towers A and C are located at the same level on opposite sides of the hill. The power line is 10 m above the ground on both sides of the hill.

The power line between the towers follows the path of the function $y = -\frac{1}{2}|x - 150| + 40$

where x represents the horizontal distance from the transformer station and
y represents the height above ground level at the transformer station.

This is shown in the diagram below.



To the nearest tenth of a metre, what is the total length of the power line connecting the three towers?

19 Mario and Tina have decided to start investing money at the same time.
A financial advisor gives them the following rule:

$$A = P(1 + r)^t$$

where
- A: accumulated money
- P: the money (principal) started with
- r: interest rate
- t: the number of years the money is invested

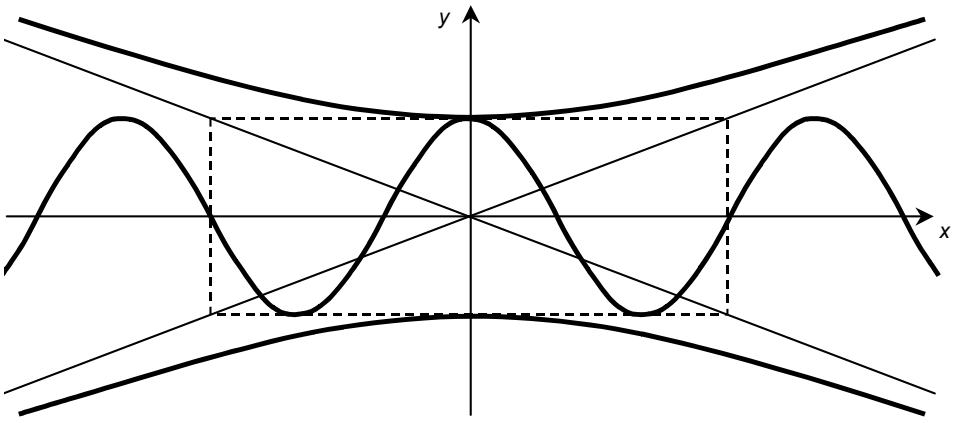
Mario is investing $5000 compounded annually at an interest rate of 5%.

Tina is investing $3000 compounded annually at 6% interest rate.

Tina calculated the time at which Mario would have doubled his original investment. She rounded the time to the nearest tenth of a year and calculated how much money she would have accumulated by that time.

How much money will Tina have accumulated by the time that Mario has doubled his investment?

20 A hyperbola and a trigonometric function are drawn on the same Cartesian plane, as shown in the diagram below. The asymptotes of the hyperbola are also shown.

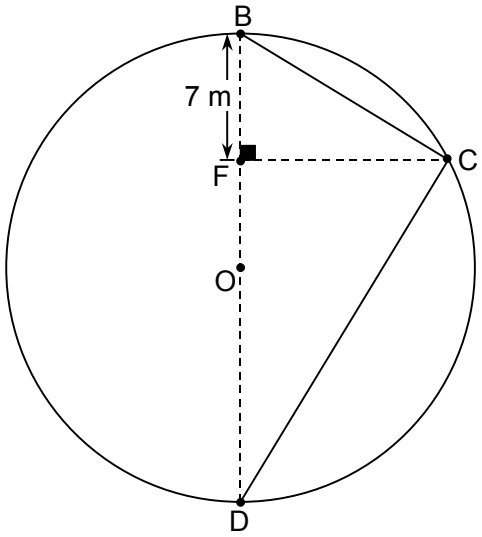


The equation of the trigonometric function is $g(x) = 4 \cos\left(\frac{\pi x}{3}\right)$.

What is the equation of the hyperbola?

21 The diagram below represents a circular pool with diameter 26 metres and centre O. There is a fountain in the pool, at point F, which is 7 metres from B, along diameter BD. Segment CF is perpendicular to segment BD.

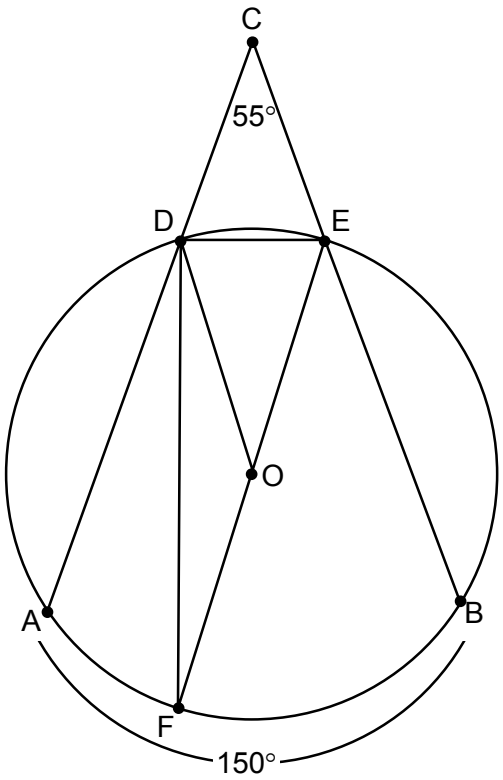
As Mary swims, she follows two straight paths that take her first from B to C and then from C to D.



Rounded to the nearest tenth of a metre, what is the total distance that Mary swims?

22 A company that builds satellite dish towers has a logo based on a circle with centre O as shown below, where:

m $\angle$ ACB = 55° and m $\overset{\frown}{AB}$ = 150°.



What is the measure of $\angle$ FDO?

Justify each step of your solution.

23 Sara attends a high school in which a prize is given to all students whose examination marks are greater than a certain Z-score. The marks (out of 100) for Sara's class are given below.

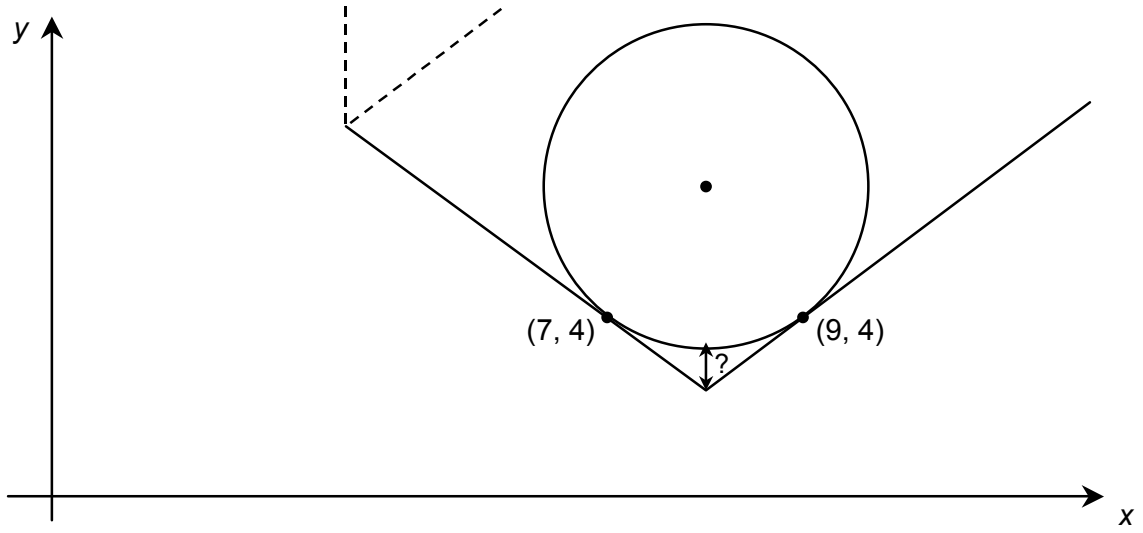
10, 52, 55, 60, 60, 62, 66, 67, 70, 72, 73, 79, 80, 81, 84, 86, 88, 91, 92, 99

Although she obtained the highest mark in the class, Sara did not get a prize. Her teacher told her that she needed to get 100 on the exam to win a prize.

Sara felt that this was unfair because one student in her class, who had left the examination room early, had obtained a very poor mark. The teacher agreed to remove the mark of 10 from the class list, but kept the original Z-score as the criteria to win a prize.

Rounded to the nearest whole number, what was the recalculated minimum mark required to win the prize?

24 A metal disc is placed in the corner of a room, as shown in the Cartesian plane below, scaled in centimetres. The disc is represented by the circle $(x - 8)^2 + (y - 6)^2 = 5$. The walls forming the corner of the room are represented by an absolute value function. The disc touches one wall at (7, 4) and the other at (9, 4).

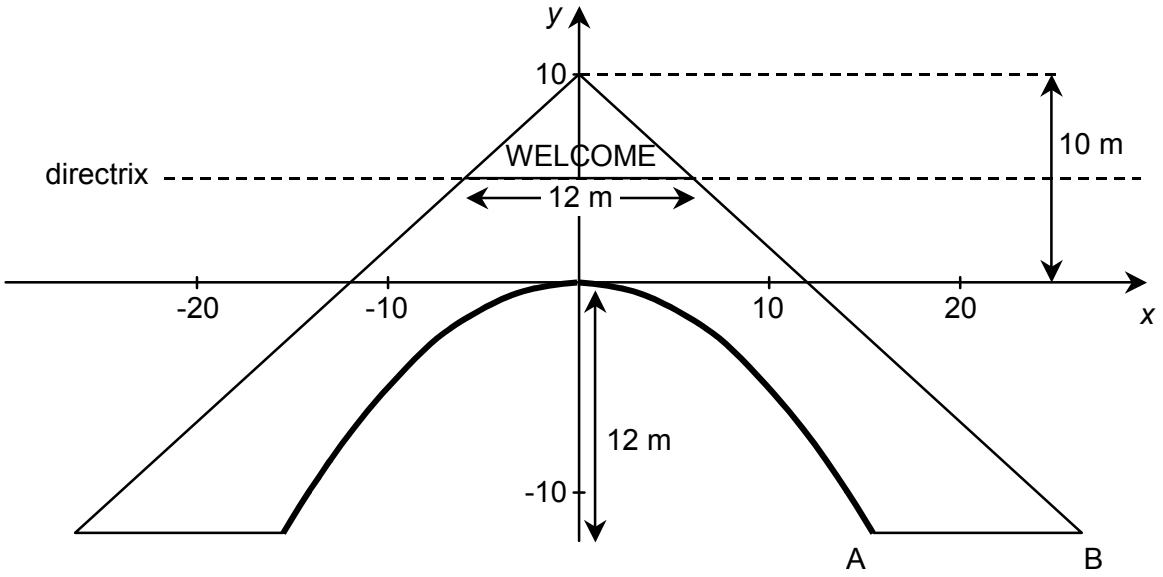


What is the minimal distance separating the disc and the corner of the room, to the nearest tenth of a centimetre?

25 A designer has suggested a massive entrance to welcome athletes to the Olympic Games in Athens in 2004 as shown in the Cartesian plane below, scaled in metres. The athletes will enter through an arch that is in the form of a parabola and is 12 m high. The parabola can be represented by the rule of correspondence $x^2 = -20y$.

The entry has the word WELCOME written on a line segment 12 m wide. The line segment is part of the directrix of the parabola.

The outer part of the entry is in the form of an absolute value function with vertex 10 m above the top of the arch.



To the nearest tenth of a metre, what is the width of the base $\overline{AB}$ of the entrance?

GRICS

568-526

Mathematics

FOR TEACHER USE ONLY	
Part A	_____ /40
Part B	_____ /24
Part C	_____/36
Total	_____ /100

Comprehensive Exam
Number 54

Answer Booklet

Secondary 5

September 2004

Student's Name	
Group	Date

Youth Sector General Education

Part A **Questions 1 to 10**

Blacken the letter that corresponds to the answer chosen.

Each question is worth 4 marks.

1	[A]	[B]	[C]	[D]	**6**	[A]	[B]	[C]	[D]
2	[A]	[B]	[C]	[D]	**7**	[A]	[B]	[C]	[D]
3	[A]	[B]	[C]	[D]	**8**	[A]	[B]	[C]	[D]
4	[A]	[B]	[C]	[D]	**9**	[A]	[B]	[C]	[D]
5	[A]	[B]	[C]	[D]	**10**	[A]	[B]	[C]	[D]

Part B **Questions 11 to 16**

Write your answer in the space provided.

11

Statement	Justification
2. m $\overline{BC} = 80°$	
3. m $\angle$ BOC = 80°	

4	2	0

12 The degree measure of arc EF is _____. | 4 | 0 |

13 The equation of the ellipse is _____. | 4 | 0 |

14 Solution(s): _____ | 4 | 2 | 0 |

15 Show all your work. | 4 | 3 | 2 | 0 |

The company had either no profit or a loss for _____ days.

16 Show all your work. | 4 | 3 | 1 | 0 |

$$\csc^2 x + \sec^2 x = \frac{1}{\sin^2 x \cos^2 x}$$

Part C Questions 17 to 25

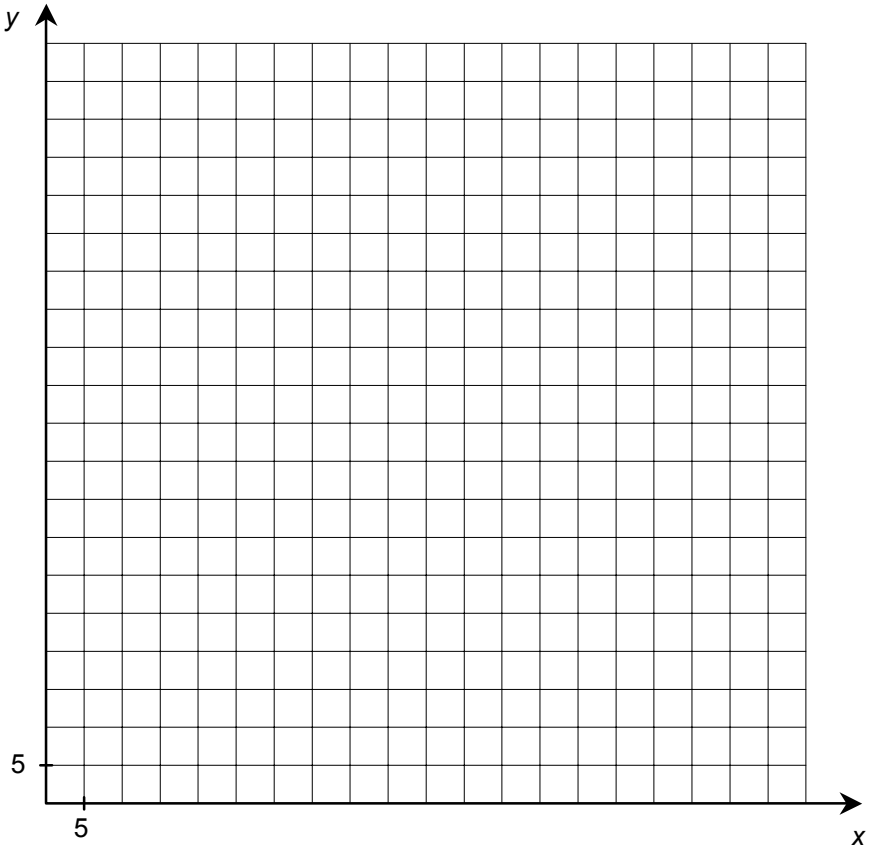
- Show all your **work** as well as your **answer**. The work shown is taken into consideration when marks are awarded.
- Your written information must be legible, complete, and clearly stated in correct language so the marker understands exactly what you have done.

Even if your answer is correct, no marks will be given unless acceptable work is shown.

17

4	3	2	1	0

Show all your work.

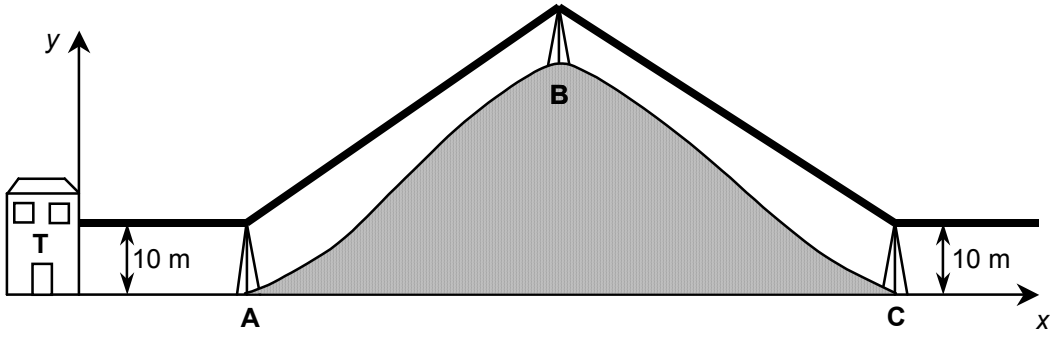


Answer: The company must sell _____ gas and _____ electric mowers to maximize its revenue.

18

4	3	2	1	0

Show all your work.



Answer: To the nearest tenth of a metre, the total length of the power line connecting the three towers is _____ m.

19

4	3	2	1	0

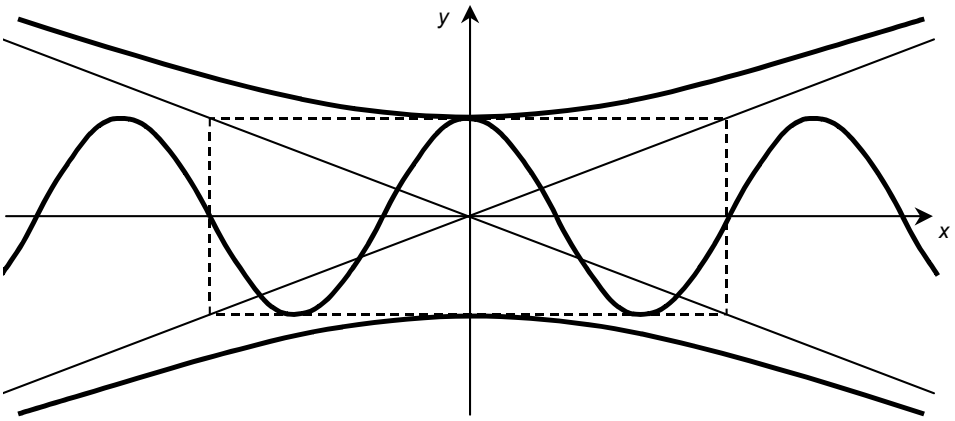
Show all your work.

Answer: By the time that Mario has doubled his investment, Tina will have accumulated $_____.

20

4	3	2	1	0

Show all your work.

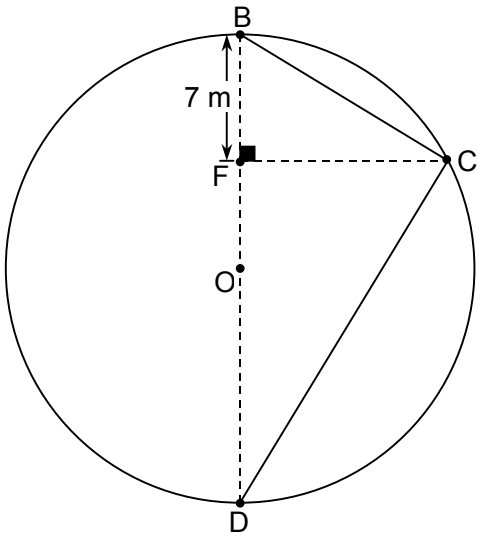


Answer: The equation of the hyperbola is _____.

21

4	3	2	1	0

Show all your work.

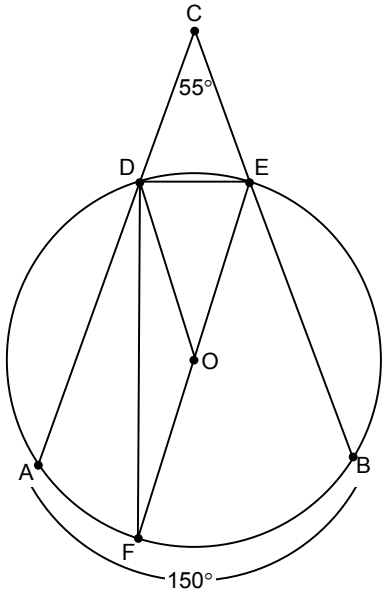


Answer: Rounded to the nearest tenth of a metre, Mary swims a total of _____ metres.

22

4	3	2	1	0

Show all your work.



Statement	Justification

Answer: The measure of $\angle$ FDO is _____.

23

4	3	2	1	0

Show all your work.

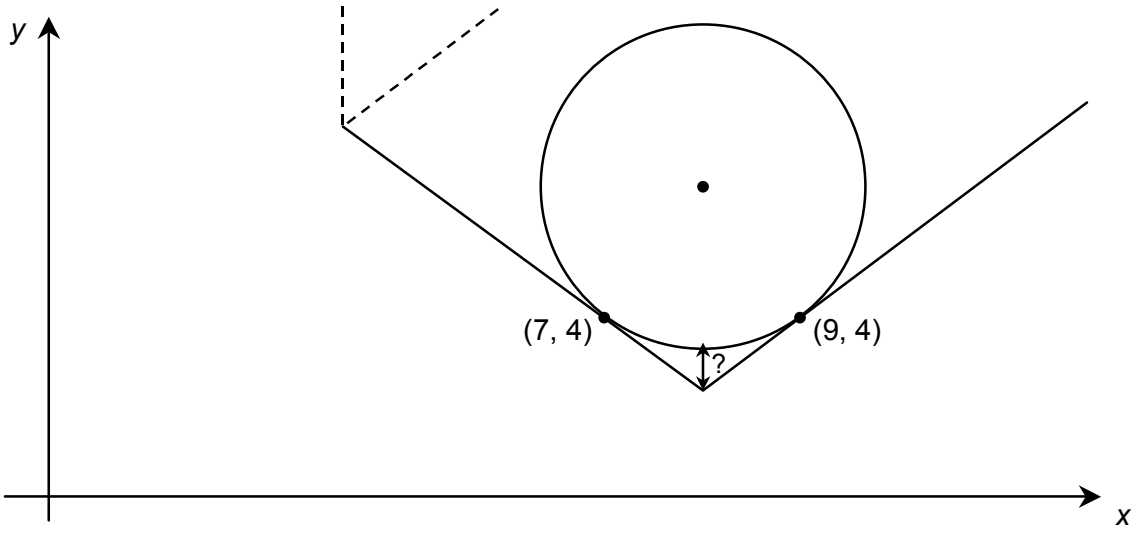
10, 52, 55, 60, 60, 62, 66, 67, 70, 72, 73, 79, 80, 81, 84, 86, 88, 91, 92, 99

Answer: The new minimum mark, rounded to the nearest whole number, required to win the prize was _____.

24

4	3	2	1	0

Show all your work.

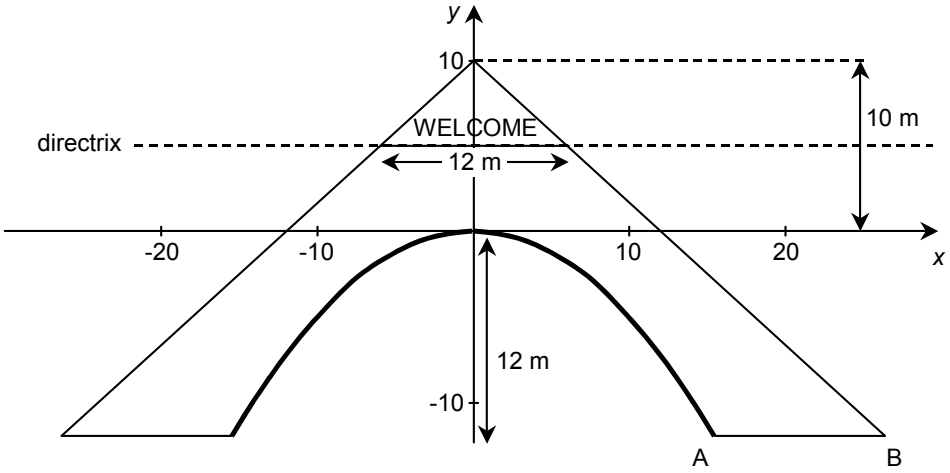


Answer: To the nearest tenth of a centimetre, the minimal distance separating the ball and the corner of the room is _____ cm.

25

4	3	2	1	0

Show all your work.



Answer: To the nearest tenth of a metre, the width of the base $\overline{AB}$ is _____