

DAWSON COLLEGE
MATHEMATICS DEPARTMENT

Final Examination
Winter 2010

Calculus II (201-203-DW)

Date: Thursday, May 20, 2010

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1. [5 marks] Find $f(x)$ if

$$f'(x) = \frac{\sqrt{x}}{2} \left(8\sqrt{x} + \frac{7}{x^{3/2}} - \frac{6}{x^2} \right)$$

and $f(1) = 1$.

2. [5 marks] The daily cost function associated with producing an item is given by the function

$$C'(x) = 0.0036x^2 - 0.08x + 99$$

where $C'(x)$ is measured in dollars / unit, and x denotes the number of units produced. The daily fixed cost incurred is 235\$. Find the total cost incurred in producing the first 150 units.

3. [5 marks] Each week, the quantity produced x (in units of a thousand) of a certain commodity is related to the unit price p (in dollars) by the demand function

$$p = -0.03x^2 - 1.2x + 21$$

If the market price is set at 6\$, find the consumers' surplus.

4. [5 marks] In 2009, the sales (in thousands of dollars) of a certain brand of shoes can be represented by the function

$$S(t) = \frac{\sqrt{t} + 50}{2}$$

where t represents the number of days elapsed since the start of the year 2009. Determine the **average value** of these sales over the interval of days 4 to 49.

5. [20 marks] Solve the following integrals:

a.

$$\int (x+1) (x-5)^{85} dx$$

b.

$$\int \left(\cos(-2x) - \sin\left(\frac{x}{5}\right) \right) dx$$

c.

$$\int 16 x^3 \ln x dx$$

d.

$$\int \frac{-9x - 18}{(x^2 + 6)(x - 3)} dx$$

6. **[5 marks]** Find the area of the region completely enclosed by the graphs of

$$y_1 = x^2 + x - 3 \quad \text{and} \quad y_2 = 4x + 1$$

7. **[5 marks]** Use the limit definition (Riemann Sums) of the definite integral to evaluate

$$\int_0^2 (-x^3 + 3x^2 + 2) dx$$

8. **[5 marks]** Use Simpson's Rule with $n = 6$ to approximate the value of the definite integral

$$\int_2^8 \sqrt{x^2 + 1} dx$$

(Round your answer to 4 decimal places.)

9. **[5 marks]** Find the fourth Taylor Polynomial of the function

$$f(x) = x^5 + 5 \ln x$$

at $a = 1$.

10. **[5 marks]** Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{x - 4 + 4e^{-x} + 3\ln(x+1)}{5 - 5\cos x}$$

11. [5 marks] Evaluate the integral, if it converges:

$$\int_{\pi}^{\infty} x e^{-x^2+1} dx$$

12. [5 marks] Verify that $y = 3x^3 + e^{-4x}$ is a solution to the differential equation

$$2y' + 8y - 24x^3 = xy'' - 16xe^{-4x}$$

13. [5 marks] Use separation of variables to find the solution of the differential equation

$$(5y - 2y^2) y' = \frac{6y}{(3x - 2)^2}$$

subject to the initial condition $y(1) = -2$.

14. [5 marks] Find the sum of the convergent series

$$\sum_{n=2}^{\infty} \frac{8(-4)^{n-1}}{7^n}$$

15. [15 marks] Determine if each of the following series is convergent or divergent. State the test used.

a.

$$\sum_{n=1}^{\infty} \frac{33 + 4n - 21n^3}{7n^3 - 6n^2 + 10}$$

b.

$$\sum_{n=1}^{\infty} \frac{1}{5 + \sqrt{n^7}}$$

c.

$$\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$$

Answers

- $f(x) = 2x^2 + \frac{7}{2} \ln x + \frac{6}{\sqrt{x}} - 7$
- $C(150) = 18\,235\$$
- $CS = 80\,000\$$
- 27.48 thousands of \$
- $\frac{(x-5)^{87}}{87} + \frac{3(x-5)^{86}}{43} + C$
 - $-\frac{1}{2} \sin(-2x) + 5 \cos\left(\frac{x}{5}\right) + C$
 - $(\ln x)(4x^4) - x^4 + C$
 - $\frac{3}{2} \ln|x^2 + 6| - 3 \ln|x - 3| + C$
- $A = 20.83 \text{ units}^2$
- 8
- 30.6798
- $P_4(x) = -9 + 10x + \frac{15}{2}(x-1)^2 + \frac{35}{3}(x-1)^3 + \frac{15}{4}(x-1)^4$
- $\frac{1}{5}$
- $\frac{1}{2} e^{-\pi^2 + 1} = 0.00007$
- Should get equality: $18x^2 = 18x^2$
- $5y - y^2 = \frac{-2}{3x-2} - 12$
- $S = \frac{-32}{77}$
- Series diverges by the divergence test (or nth term test), since $\lim_{n \rightarrow \infty} a_n = -3 \neq 0$
 - Series converges by the comparison test (compare to the converging p-series with $p = \frac{7}{2} > 0$)

c) Series converges by the integral test, since $\int_2^{\infty} \frac{2}{x(\ln x)^3} dx = 2.08$