Dawson College: Department of Mathematics
Final Examination: Winter 2010: 201-NYB-05: Calculus II (SCIENCE)

Question 1. Evaluate the indefinite integrals:

a. *(5 marks)*

$$\int \left(x^2+x\right)\sin\left(2x^3+3x^2\right)\,dx$$

b. *(5 marks)*

$$\int \ln\left(x^2+4\right)\,dx$$

Answer: a. $\frac{-1}{6}\cos(2x^3+3x^2)+C$ b. $x\ln(x^2+4)-2x+\arctan\left(\frac{x}{2}\right)+C$

Question 2. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{1}{t^2\sqrt{9t^2+1}}\,dt$$

Answer: $-\frac{\sqrt{9t^2+1}}{t}+C$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{2x^2+x+1}{x^3+x}\,dx$$

Answer: $\ln|x|+\frac{1}{2}\ln(x^2+1)+\arctan x+C$

Question 4. *(5 marks)* Evaluate the definite integral:

$$\int_0^3 \frac{x-\sqrt{x+1}}{\sqrt{x+1}}\,dx$$

Answer: $\frac{-1}{3}$

Question 5. *(5 marks)* Find and simplify:

$$\frac{d}{dx}\left[\int_{\arcsin x}^{2x} \sqrt[3]{\sin t}\,dt\right]$$

Answer: $2\sqrt[3]{\sin(2x)}-\frac{\sqrt[3]{x}}{\sqrt{1-x^2}}$

Question 6. *(5 marks)* Use only the definition of the definite integral to evaluate:

$$\int_1^2 (1-6x^2)\,dx.$$

Answer: -13

Question 7. *(5 marks)* Find the average value of the function

$$f(x) = \frac{x^5 + x^3 + x}{x^2 + 9}$$

over the interval $[-3, 3]$.

Answer: 0

Question 8. *(5 marks)* Given

$$\int_a^b g(x)\,dx = 6, \quad \int_a^c g(x)\,dx = 3, \quad \int_c^a h(x)\,dx = -1, \quad \int_c^b h(x)\,dx = 11,$$

find

$$\int_a^b (2g(x) - h(x))\,dx.$$

Answer: 0

Question 9. *(5 marks)* Find the total area of the region bounded by the graphs of $y = \frac{3}{x}$, $y = 2x - 1$ between $x = 1$ and $x = e$.

Answer: $6\ln\left(\frac{3}{2}\right) + e^2 - e - \frac{9}{2}$

Question 10. *(5 marks)* Find the volume of the solid obtained when the region bounded by the graphs of $f(x) = x^2$ and $g(x) = \sqrt{x}$ is rotated about the line $x = -1$.

Answer: $\frac{29}{30}\pi$

Question 11. *(5 marks)* A hemispherical bowl is filled with liquid chocolate which has a density of $\rho = 1200\frac{kg}{m^3}$. If the bowl is $0.26m$ across the top *(diameter)*, how much work is required to drink the entire bowl of chocolate through a straw that extends $0.20m$ above the top edge? *($g = 9.8\frac{m}{s^2}$)*

Answer: ≈ 13.4604J

Question 12. *(5 marks)* Find the arc length of the graph of the function

$$y = \ln(\cos x)$$

over the interval $\left[0, \frac{\pi}{3}\right]$.

Answer: $\ln\left(2 + \sqrt{3}\right)$

Question 13. Evaluate the limits:

a. *(5 marks)*

$$\lim_{x\to\infty} x\left(\frac{\pi}{2} - \arctan(x)\right)$$

b. *(5 marks)*

$$\lim_{x\to 0^+} (\sin x)^x$$

Answer: a. 1   b. 1

Question 14. *(5 marks)* Evaluate the improper integral or show it diverges:

$$\int_{e^2}^{\infty} \frac{\ln x}{x^2}\,dx$$

Answer: $\frac{3}{e^2}$

Question 15. *(5 marks)* Find the sum of the following series if it converges or show it diverges.

$$\sum_{n=3}^{\infty} \frac{1}{(3n-2)(3n+1)}$$

Answer: $\frac{1}{21}$

Question 16. Determine whether each of the following series converges or diverges. Justify your answer.

a. *(5 marks)*

$$\sum_{n=10}^{\infty} \frac{\sqrt{n^9+n^4-1}}{\sqrt{n^9+1}}$$

b. *(5 marks)*

$$\sum_{n=1}^{\infty} \frac{e^{-n^{1/3}}}{n^{2/3}}$$

Answer: a. Diverges, by the test for divergence   b. Converges, by the integral test

Question 17. Determine whether each of the following series converges or diverges. Justify your answer.

a. *(5 marks)*

$$\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$$

b. *(5 marks)*

$$\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{2} - \frac{1}{n}\right)$$

Answer: a. Converges, by the ratio test   b. Diverges, by the limit comparison test

Question 18. *(5 marks)* Find the Taylor Polynomial of order 3 of $f(x) = x^2 \sin x$ at $x = \pi$.

Answer: $P_3(x) = -\pi^2(x-\pi) - 2\pi(x-\pi)^2 + \left(\frac{1}{6}\pi^2 - 1\right)(x-\pi)^3$