

**DAWSON COLLEGE**  
**Mathematics Department**  
**Final Examination – WITH ANSWERS**  
**Calculus II**  
**201-NYB-05 (REGULAR)**  
**May 26, 2010**

1. [28 marks]. Evaluate the following integrals

a)  $\int x \sin(2x) dx$  (4 Marks)

$$-\frac{1}{2}x \cos(2x) + \frac{1}{4} \sin(2x) + c$$

b)  $\int \frac{x}{(x-1)(x+1)^2} dx$  (6 Marks)

$$\frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| - \frac{1}{2(x+1)} + c$$

c)  $\int \tan(4x) \sec^3(4x) dx$  (4 Marks)

$$\frac{1}{12} \sec^3(4x) + c$$

d)  $\int \frac{\sqrt{x}-1}{\sqrt{x}+1} dx$  (4 Marks)

$$(\sqrt{x}+1)^2 - 6(\sqrt{x}+1) + 4 \ln|\sqrt{x}+1| + c$$

$$e) \int \frac{1}{x^2 \sqrt{4+x^2}} dx$$

(6 Marks)

$$-\frac{\sqrt{4+x^2}}{4x} + c$$

$$f) \int \frac{\sin^3 x}{\sqrt{\sec x}} dx$$

(4 Marks)

$$\frac{2}{7} \cos^{\frac{7}{2}} x - \frac{2}{3} \cos^{\frac{3}{2}} x + c$$

2. (5 Marks) Calculate the definite integral  $\int_0^2 (3x^2 + 1) dx$  using

a) the definition of the definite integral (i.e. Riemann Sums)

(4 Marks)

b) the Fundamental Theorem of Calculus

(1 Mark)

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**NOTE:**

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

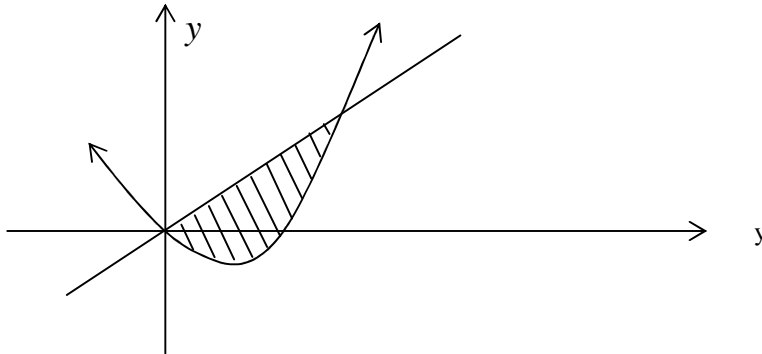
$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

(a) 10

**Answer:**

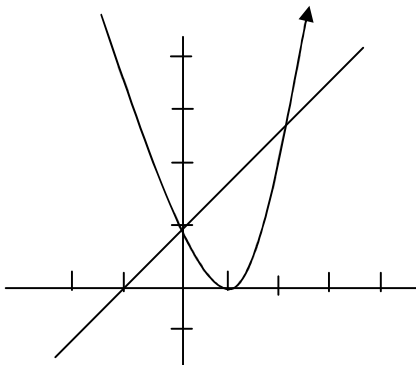
(b) 10

3. (5 Marks) Find the area of the region bounded by the curves  $y = x^2 - x$  and  $y = 3x$  as shown in the figure.



Answer:  $\frac{32}{3}$

4. (10 Marks) Find the volume of the solid generated when the region enclosed by the graphs of the functions  $y = (x-1)^2$  and  $y = x+1$  is revolved
- about the  $x$ -axis
  - about the  $y$ -axis



Answer:

(a)  $\frac{72\pi}{5}$

(b)  $\frac{27\pi}{2}$

5. (5 Marks) Find the arc length of the graph of  $y = \ln(\sec x)$ ,  $0 \leq x \leq \frac{\pi}{4}$

$$\ln(\sqrt{2} + 1)$$

6. (5 Marks) Find the average value of  $f(x) = x(x+1)^{\frac{1}{2}}$  on the interval  $[0, 3]$

$$\frac{116}{45}$$

7. (8 Marks) Calculate the following limits

a)  $\lim_{x \rightarrow 1} \frac{\ln x - \sin(x-1)}{(x-1)^2}$

$$-\frac{1}{2}$$

b)  $\lim_{x \rightarrow 0} (1+2x)^{\frac{3}{x}}$

$$e^6$$

8. (8 Marks). Calculate each improper integral or show that it diverges

a)  $\int_e^{\infty} \frac{dx}{x(\ln x)^{\frac{2}{3}}}$

**Diverges**

b)  $\int_1^{\infty} \frac{e^x dx}{1+e^{2x}}$

$$\frac{\pi}{2} - \tan^{-1}(e)$$

9. (5 Marks) Find the sum of the infinite series

$$\sum_{n=0}^{\infty} \left(\frac{2}{\pi}\right)^{n+1}$$

$$\frac{2}{\pi-2}$$

10. (16 Marks) Determine whether the following series converge or diverge

a)  $\sum_{n=1}^{\infty} \frac{4^{n+1}}{(2n)!}$

**Converges by Ratio Test**

b)  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3 + 2n - 1}$

**Converges by Limit Comparison Test**

c)  $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

**Converges by Integral Test**

d)  $\sum_{n=1}^{\infty} \frac{3n^2 + 7n + 7}{10 + 2n^2 + 5n}$

### Diverges by $n^{\text{th}}$ term Test

11. (5 Marks) Find the Maclaurin polynomial of degree 3 for the function  $f(x) = e^x \sin x$

$$x + x^2 + \frac{1}{3}x^3$$